Federal Reserve Bank of Philadelphia

Working Papers
Research Department

WP 19-12
February 2019
https://doi.org/10.21799/frbp.wp.2019.12

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ISSN: 1962-5361

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January 2019

Abstract

We construct a model of consumer credit with payment frictions, such as spatial separation and unsynchronized trading patterns, to study optimal monetary policy across different interbank market structures. In our framework, intermediaries play an essential role in the functioning of the payment system, and monetary policy influences the equilibrium allocation through the interest rate on reserves. If interbank credit markets are incomplete, then monetary policy plays a crucial role in the smooth operation of the payment system. Specifically, an equilibrium in which privately issued debt claims are not discounted is shown to exist provided the initial wealth in the intermediary sector is sufficiently large relative to the size of the retail sector. Such an equilibrium with an efficient payment system requires setting the interest rate on reserves sufficiently close to the rate of time preference.

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Keywords: Intermediation, liquidity, payments system, rediscounting

JEL classifications: E42, E58, G21

1. INTRODUCTION

A prominent financial institution of modern economies is the payments system, a formal arrangement among market participants designed to facilitate the repayment of private debt claims. An important feature of these systems is the presence of financial intermediaries buying and selling debt claims originated in retail transactions and finally clearing all debts. These intermediaries usually operate by holding reserves that serve as an essential instrument to help them achieve a desired portfolio of private claims. Because of intermediaries' active role in clearing private claims and their demand for reserves, many economists have highlighted the important role of monetary policy in helping the adequate functioning of the economy's payment system.

A widely held view among monetary economists is that the payments system works efficiently when it is satiated with liquidity (i.e., when reserves are plentiful for intermediaries involved in the clearing of payment instruments). This view is based on the Friedman rule, which provides a rationale for the optimum quantity of money in the economy.[1] The Friedman rule asserts that optimal monetary policy should aim at eliminating the opportunity cost of holding money balances, such as reserves held at the central bank, for transaction purposes. This monetary policy prescription has been shown to be optimal in a variety of economic environments. Woodford (1990) and Williamson and Wright (2010a,b) provide comprehensive surveys of optimal monetary policy across different monetary environments.

Absent in the majority of those papers is the explicit modeling of payment and settlement imperfections, such as spatial separation, unsynchronized trading patterns, and imperfect information. A notable exception is Freeman (1996a), who first proposed a dynamic general equilibrium framework with explicit payment frictions to study how monetary policy affects the settlement of private debt claims. Some subsequent important contributions to the payments literature include Freeman (1996b), Kahn and Roberts (1998), Williamson

[1] For more on this principle, see Friedman (1959, 1969).

(1998), Temzelides and Williamson (2001), Martin (2004), and Mills (2006).[2] Papers in this literature explicitly consider the underlying payment frictions agents face when trading in the marketplace and study the role of government policies to achieve efficiency in the payment system. The main drawback of this literature is that the underlying framework is highly specialized to account for the payment frictions, making it difficult to relate to other monetary models.

Our goal in this paper is to propose a unified framework that explicitly considers both monetary and payment frictions to study optimal monetary policy. We believe that such a framework will allow us to shed some light on the links (and potential trade-offs) between monetary policy and the efficient operation of the payments system. We do so in a framework that can be easily compared with other papers in the literature that study optimal monetary policy without payment frictions. As a result, we can answer the following research questions: What is the role of interbank credit markets in the functioning of the payment system? Does the efficient operation of the payment system require complete interbank credit markets? Is it possible to attain an efficient payment system if markets are incomplete? If so, what kinds of monetary policies are consistent with efficiency? Does the Friedman rule lead to efficiency in the payment system regardless of the interbank market structure?

In the framework developed in this paper, consumers buy goods from merchants by issuing debt claims. Unsynchronized trading patterns in the environment imply that consumers and merchants do not get together for the settlement of debts. A subset of agents, referred to as intermediaries, has the ability to sequentially interact with merchants and consumers, respectively, so that they can buy debt claims from merchants and subsequently retire them by directly contacting the issuers. Our analysis considers a credit system in which financial intermediaries play a crucial role in the clearing of debt claims originated in retail transactions, giving rise to a competitive rediscounting market.

The timing of events within the period is such that intermediaries must hold a portfolio

[2] Kahn and Roberds (2009) provide a comprehensive survey of the payments literature. Nosal and Rocheteau (2017) provide a useful textbook analysis of the economics of payments.

of liquid assets to transfer their wealth across periods to take advantage of rediscounting opportunities. The government supplies reserves that serve as a store of value for intermediaries, permitting the smooth functioning of the payments system.[3] The supply of reserves influences the amount of funds flowing into the rediscounting market, which determines the prevailing discount rate. By setting the interest rate on reserves the central bank affects the functioning of the intermediated credit system through the discount rate on private debt claims. Given this channel of transmission, we study the links between monetary policy and the efficient functioning of the payment system.

To help answer our research questions, we consider two distinct market structures. First, we consider an environment with incomplete interbank markets to verify whether monetary policy, through the interest rate on reserves, can enhance the functioning of the payments system in the absence of interbank exchange. We then analyze an environment with complete interbank markets and show that the existence of a perfect credit interbank market is sufficient to overcome the payment frictions, resulting in an efficient payment system.

When interbank credit markets are incomplete, we show that it is possible to select the interest rate on reserves sufficiently close to the rate of time preference to obtain an equilibrium in which privately issued debt claims are not discounted, resulting in an efficient payment system. This efficient payment arrangement is obtained only if the fundamentals of the economy are such that the initial wealth in the intermediary sector is sufficiently large relative to the size of the retail sector. Otherwise, an equilibrium without discounting does not exist. Additionally, we show that, when an equilibrium without discounting does exist, the Friedman rule results in an efficient allocation by perfectly smoothing consumption for intermediaries. In other words, efficiency requires not only that intermediaries earn zero profits in the rediscounting business but also that reserves allow intertemporal exchange to

[3] In our analysis, money and credit are complements, as opposed to many papers in the literature in which the underlying equilibrium implies that money and credit are substitutes in payments, such as Monnet and Roberds (2008), Sanches and Williamson (2010). Gomis-Porqueras and Sanches (2013) consider a model in which money and credit are complements. In that framework it is shown that the existence of a credit system increases the set of feasible government policies. So the complementarity between money and credit is due to the implementation of policies, not a market arrangement to improve payments.

occur smoothly, which can be attained by setting the interest rate on reserves equal to the rate of time preference.

When interbank credit markets are complete, we then show that an equilibrium without discounting always exists. That is, regardless of the underlying fundamentals of the economy, complete markets are sufficient to overcome the payment frictions in the model so that there is an equilibrium in which the payment system is efficient. As in the previous case, efficiency in the overall allocation of resources in the economy requires the implementation of the Friedman rule to eliminate the opportunity cost of holding reserves across periods and allow intermediaries to attain perfect consumption smoothing.

Given these results, one can conclude that, in the absence of complete interbank credit markets, monetary policy is an essential tool to guarantee the smooth functioning of the payment system. The central bank can use the interest rate on reserves to influence the discount rate in the rediscounting market for debt claims, where intermediaries buy and sell claims for a profit. Monetary policy cannot always implement an efficient allocation when interbank markets are incomplete, but it can enhance the functioning of the payment system by lowering the discount rate and increasing output in decentralized markets. Finally, we argue that promoting entry into the rediscounting business leads to an efficient payment system because it reduces the discount rate, holding other factors constant.

The rest of the paper is organized as follows. Section 2 describes the basic framework. Section 3 characterizes efficient allocations by solving the planner's problem. In section 4, we study equilibrium allocations under incomplete interbank credit markets. In section 5, we characterize equilibrium allocations under complete markets. Section 6 provides a discussion of the main results. Section 7 concludes.

2. MODEL

Our basic framework builds on Lagos and Wright (2005) and Rocheteau and Wright (2005). Time is discrete and continues forever. Each period is divided into two subperiods in which economic activity will differ. There is a frictionless centralized market (CM) in the first subperiod, while trade is decentralized (DM) in the second subperiod. A perishable consumption good is produced and consumed in each subperiod. We refer to the consumption good produced in the first subperiod as the CM good and to the consumption good produced in the second subperiod as the DM good.

There are three types of agents, referred to as consumers, merchants, and bankers. Consumers and merchants are both infinitely lived, with a $[0,1]$-continuum of each type. A banker lives for two consecutive periods only. In each period, a measure $\alpha \in \mathbb{R}_+$ of new bankers is born. In the initial period, there is a measure α of old bankers. All agents discount future periods at the rate $\beta \in (0,1)$.

The production possibilities in the economy are as follows. Each consumer has access to a divisible technology that allows him to produce one unit of the CM good with one unit of effort. Each merchant has access to a divisible technology that allows him to produce one unit of the DM good with one unit of effort. A banker does not have access to any production technology but receives an endowment of $e \in \mathbb{R}_+$ units of the CM good in the first period of his life.

The consumer's preferences are represented by

$$U^c\left(x_t^c, q_t^c\right) = x_t^c + u\left(q_t^c\right),$$

where $x_t^c \in \mathbb{R}$ is net consumption of the CM good and $q_t^c \in \mathbb{R}_+$ is consumption of the DM good. The function $u : \mathbb{R}_+ \to \mathbb{R}$ is twice continuously differentiable, increasing, and strictly concave, with $u(0) = 0$ and $u'(0) = \infty$. The merchant's preferences are represented by

$$U^m\left(x_t^m, q_t^m\right) = x_t^m - w\left(q_t^m\right),$$

where $x_t^m \in \mathbb{R}_+$ is consumption of the CM good and $q_t^m \in \mathbb{R}_+$ is production of the DM good. Assume that $w : \mathbb{R}_+ \to \mathbb{R}_+$ is continuously differentiable, increasing, and convex, with $w(0) = 0$. Assume that $u'(q)/w'(q)$ is strictly decreasing and that $\lim_{q\to 0} u'(q)/w'(q) = \infty$.

The banker's preferences are represented by

$$U^b\left(x_t^y, x_{t+1}^o\right) = v\left(x_t^y\right) + \beta v\left(x_{t+1}^o\right),$$

where $x_t^y \in \mathbb{R}_+$ denotes consumption in the first period of the CM good and $x_{t+1}^o \in \mathbb{R}_+$ denotes consumption in the second period. The function $v : \mathbb{R}_+ \to \mathbb{R}$ is continuously differentiable, increasing, and concave. Finally, assume $w(q^*) \leq \alpha\beta e$.

The physical environment where the agents interact is as follows. There exist two distinct locations in the economy, referred to as the bankers' location and the central location. Consumers and merchants visit the central location in the first subperiod. An important characteristic of the environment is that consumers and merchants do not overlap in the central location. As in Freeman (1996b), we assume that merchants arrive first and depart before all consumers arrive.

In the second subperiod, a consumer is randomly and bilaterally matched with a merchant with probability one. Following the literature, we refer to the collection of bilateral meetings in the second subperiod as the decentralized market.

A banker is born in the bankers' location in the first period of his life. In the second period, he visits the central location, returning to the bankers' location before the end of the period. Bankers can transport goods at no cost from the bankers' location to the central location and vice versa.

3. EFFICIENT ALLOCATIONS

We start by solving the social planner's problem. We can, without loss of generality, restrict attention to stationary allocations. Additionally, we assume equal weights across all generations. The planner chooses an allocation

$$(x_t^c, x_t^m, x_t^y, x_t^o, q_t) \in \mathbb{R} \times \mathbb{R}_+^4$$

in every period t to maximize the joint surplus of infinitely-lived agents

$$\sum_{t=0}^{\infty} \beta^t \left[x_t^m + x_t^c + u(q_t) - w(q_t)\right]$$

subject to the feasibility conditions

$$\alpha x_t^y + \alpha x_t^o + x_t^m + x_t^c = \alpha e, \tag{1}$$

$$x_t^m \le \alpha e, \tag{2}$$

and the banker's participation constraint

$$v\left(x_t^y\right) + \beta v\left(x_{t+1}^o\right) \ge U^b, \tag{3}$$

with $U^b \in \mathbb{R}_+$ exogenously given. Constraint (2) arises because of the spatial separation in the environment and reminds us that the per capita amount of CM output that can be allocated to merchants cannot exceed the per capita endowment of bankers.

Let $\eta_t \in \mathbb{R}_+$ denote the multiplier on (1), $\delta_t \in \mathbb{R}_+$ the multiplier on (2), and $\alpha\lambda_t \in \mathbb{R}_+$ the multiplier on (3). We can summarize the first-order conditions for the planner's problem as

$$1 = \eta_t + \delta_t,$$

$$\lambda_t v'\left(x_t^y\right) = \eta_t, \tag{4}$$

$$\lambda_t v'\left(x_{t+1}^o\right) = \eta_{t+1}, \tag{5}$$

$$u'\left(q_t\right) = w'\left(q_t\right), \tag{6}$$

together with the complementary slackness conditions

$$\delta_t\left(x_t^m - \alpha e\right) = 0$$

and

$$\lambda_t\left[v\left(x_t^y\right) + \beta v\left(x_{t+1}^o\right) - U^b\right] = 0.$$

Condition (6) implies

$$q_t = q^*$$

at all dates, with q^* denoting the surplus-maximizing quantity (i.e., q^* is the unique solution to $u'(q) = w'(q)$). If we restrict attention to stationary solutions, we can describe an allocation by a stationary plan $(x^c, x^m, x^y, x^o, q) \in \mathbb{R} \times \mathbb{R}_+^4$. Then, the first-order conditions (4) and (5) imply

$$x^y = x^o = x^*.$$

In other words, the planner chooses an allocation with perfect consumption smoothing for each banker.

It is straightforward to show that (3) is binding. Then, we obtain the optimal consumption allocation for bankers:

$$x^* = v^{-1}\left(\frac{U^b}{1+\beta}\right).$$

Finally, consumption plans for merchants and consumers satisfy

$$x^m + x^c = \alpha\left[e - 2v^{-1}\left(\frac{U^b}{1+\beta}\right)\right]$$

with

$$x^m \leq \alpha e.$$

This concludes the description of an efficient allocation in our environment, given an exogenously set level of utility U^b for the bankers.

4. INTERMEDIATED CREDIT SYSTEM

To understand how the intermediated economy works, start with the bilateral meetings in the decentralized market. We consider a credit system in which a consumer purchases goods from a merchant by issuing a debt claim in the decentralized market. Because consumers and merchants do not overlap in the central location in the following period, the settlement of privately issued debt claims has to be intermediated by bankers.

An old banker can purchase privately issued debt claims from merchants while visiting the central location, giving rise to a rediscounting market for privately issued debt. The claims can be redeemed in the central location upon the arrival of consumers. A banker who wants to rediscount claims in the central location must save in the first period of his life to purchase debt claims in the second. The difference between the face value of a debt claim and the discount at which the debt is purchased in the rediscounting market is the banker's profits per unit of debt.

Because the merchant knows that there is a market for rediscounting privately issued debt claims in the following period, he or she is willing to produce the DM good today in exchange for a consumer's debt claim. The price at which debt claims trade in the rediscounting market influences the amount of the DM good the merchant is willing to produce. Throughout the paper, we assume that all agents have access to a technology that permits them to perfectly identify the debt claims issued by a consumer so that counterfeiting will not be a problem. Additionally, we assume that consumers can fully commit to redeem previously issued debt claims so that default is not possible.

We can interpret the first and second periods of a banker's life cycle as follows. In the first period, each banker receives his endowment and uses part of it to set up a bank. The initial capital of the bank is invested in reserves (or other assets), which will allow the bank to rediscount private debt at a profit in the following period.

In the bankers' location, there is a perfectly competitive market for reserves in which young and old bankers trade at the real price $\phi_t \in \mathbb{R}_+$ (in terms of the CM good). Let $\bar{M}_t \in \mathbb{R}_+$ denote the per-capita supply of reserves issued by the central bank in period t. In the central location, there is a perfectly competitive rediscounting market in which old bankers and merchants trade privately issued debt claims at the real price $\rho_t \in \mathbb{R}_+$. Let $n_{t+1} \in \mathbb{R}_+$ denote the per-capita supply of private debt claims. Specifically, a unit of debt issued at date t is a promise to pay one unit of the CM good at date $t+1$.

4.1. Bankers

We start the analysis by formulating and solving the banker's problem. The banker chooses a consumption profile, reserve holdings, and the amount of rediscounting to maximize utility

$$\max_{\left(x_t^y, x_{t+1}^o, M_t, \hat{n}_{t+1}\right) \in \mathbb{R}_+^4} \left[v\left(x_t^y\right) + \beta v\left(x_{t+1}^o\right)\right]$$

subject to the first-period budget constraint

$$x_t^y + \phi_t M_t \leq e,$$

the second-period budget constraint

$$x_{t+1}^o + \rho_{t+1} \hat{n}_{t+1} \leq \phi_{t+1} \left(1 + i_t\right) M_t + \hat{n}_{t+1} + \tau_{t+1},$$

and the liquidity constraint

$$\rho_{t+1}\hat{n}_{t+1} \leq \phi_{t+1}\left(1+i_t\right)M_t + \tau_{t+1}.$$

Here $M_t \in \mathbb{R}_+$ denotes reserve holdings in period t, $\hat{n}_{t+1} \in \mathbb{R}_+$ is the amount of debt claims the banker decides to purchase at date $t+1$, $1+i_t \in \mathbb{R}_+$ is the interest rate on reserves, and $\tau_{t+1} \in \mathbb{R}$ is the real value of transfers from the government at date $t+1$. The liquidity constraint is the key constraint in the banker's problem. To rediscount debt claims at a profit in the second period of his life cycle, the banker must accumulate reserves in the first period to sell them to young bankers in the second period so that he has real funds to purchase debt claims in the rediscounting market.

Let $\beta\lambda_{t+1} \in \mathbb{R}_+$ denote the Lagrange multiplier on the liquidity constraint. The corresponding first-order conditions are given by

$$-\phi_t v'\left(x_t^y\right) + \beta\left(1+i_t\right)\phi_{t+1}v'\left(x_{t+1}^o\right) + \lambda_{t+1}\beta\left(1+i_t\right)\phi_{t+1} = 0,$$

$$\left(1-\rho_{t+1}\right)v'\left(x_{t+1}^o\right) - \lambda_{t+1}\rho_{t+1} = 0,$$

$$x_t^y = e - \phi_t M_t,$$

$$x_{t+1}^o = \phi_{t+1}\left(1+i_t\right)M_t + \left(1-\rho_{t+1}\right)\hat{n}_{t+1} + \tau_{t+1}.$$

Additionally, we have the complementary slackness condition:

$$\lambda_{t+1}\left[\rho_{t+1}\hat{n}_{t+1} - \phi_{t+1}\left(1+i_t\right)M_t - \tau_{t+1}\right] = 0.$$

The first-order conditions can be combined to obtain the Euler equation:

$$\phi_t v'\left(x_t^y\right) = \beta\frac{\left(1+i_t\right)\phi_{t+1}}{\rho_{t+1}}v'\left(x_{t+1}^o\right). \tag{7}$$

The left-hand side of (7) gives the marginal cost of an additional unit of reserves at date t. The banker gives up consumption at date t to increase his balances at the real price ϕ_t so that he can rediscount debt claims at date $t+1$. An additional unit of reserves at date $t+1$ allows him to purchase $\left(1+i_t\right)\phi_{t+1}/\rho_{t+1}$ extra units of debt claims, increasing his consumption at date $t+1$.

Note that a banker obtains old-age income from two sources: the real return on reserves held across periods and the profits from rediscounting debt claims in the second period. Thus, the effective return on asset holdings is given by $\frac{(1+i_t)\phi_{t+1}}{\phi_t \rho_{t+1}}$.

We can also use (7) to derive the real price of debt claims in the rediscounting market at date $t+1$ as

$$\rho_{t+1} = \beta \frac{(1+i_t)\,\phi_{t+1}}{\phi_t} \frac{v'\left(x^o_{t+1}\right)}{v'\left(x^y_t\right)}.$$

4.2. Government

The government's budget constraint is

$$\phi_t \bar{M}_t = \phi_t \left(1 + i_{t-1}\right) \bar{M}_{t-1} + \tau_t,$$

where $\tau_t \in \mathbb{R}$ denotes the real value of transfers to old bankers. We assume that the government intervenes at the end of each period to maintain the supply of reserves constant over time. In this case, the money supply follows the law of motion

$$m_t = \frac{\phi_{t-1}}{\phi_t} m_{t-1},$$

where $m_t \in \mathbb{R}_+$ denotes real balances at date t.

4.3. Consumers and Merchants

Consider the consumer's problem in the decentralized market. Let $n_{t+1} \in \mathbb{R}_+$ denote the amount of debt the consumer issues at date t in exchange for $q_t \in \mathbb{R}_+$ units of the DM good. Let $V^c_t \in \mathbb{R}_+$ denote the consumer's lifetime utility in period t *after* retiring debt claims issued in the previous period. The Bellman equation for the consumer is

$$V^c_t = u\left(q_t\right) + \beta\left(-n_{t+1} + V^c_{t+1}\right).$$

If we denote the merchant's lifetime utility at date t by $V^m_t \in \mathbb{R}$, then his Bellman equation can be written as

$$V^m_t = -w\left(q_t\right) + \beta\left(\rho_{t+1} n_{t+1} + V^m_{t+1}\right).$$

Note that V_t^m is the merchant's lifetime utility in period t after receiving payment for the debt claims he holds. As we have seen, bankers have to be willing to intermediate the settlement of private debt claims because consumers and merchants do not overlap in the central location to directly settle debt claims. For this reason, the value of a debt claim for the merchant is given by $\rho_{t+1}n_{t+1}$.

The consumer is willing to trade in the decentralized market if the terms of trade $(q_t, n_{t+1}) \in \mathbb{R}_+^2$ satisfy

$$u(q_t) - \beta n_{t+1} \geq 0.$$

The merchant is willing to trade if

$$-w(q_t) + \beta \rho_{t+1} n_{t+1} \geq 0.$$

We assume that the consumer makes a take-it-or-leave-it offer to the merchant in the decentralized market. Given this bargaining protocol, the terms (q_t, n_{t+1}) are determined by solving the following problem:

$$\max_{(q_t, n_{t+1}) \in \mathbb{R}_+^2} \left[u(q_t) - \beta n_{t+1}\right]$$

subject to

$$-w(q_t) + \beta \rho_{t+1} n_{t+1} \geq 0.$$

The first-order conditions are

$$\frac{u'(q_t)}{c'(q_t)} = \frac{1}{\rho_{t+1}}$$

and

$$w(q_t) = \beta \rho_{t+1} n_{t+1}.$$

The amount of goods the consumer gets from the merchant depends on the discount rate that is expected to prevail in the following period. If there is no discounting (i.e., $\rho_{t+1} = 1$) the consumer gets the surplus-maximizing quantity q^* in exchange for debt claims with real value $\beta^{-1}w(q^*)$. If the discount rate is zero, the consumer credit market works smoothly, allowing consumers to purchase the surplus-maximizing quantity on credit. In other words, in the absence of discounting, we obtain efficient decentralized exchange even though the functioning of the credit system requires intermediation by profit-maximizing agents.

If $\rho_{t+1} < 1$, then the consumer gets less than the surplus-maximizing quantity, so DM output is suboptimal. Each unit of debt issued by the consumer is worth less than one unit of the CM good for the merchant when the discount rate is positive. Because the merchant anticipates a positive discount rate in the following period, he is willing to trade only if the disutility of production is less than that of the surplus-maximizing quantity.

4.4. Equilibrium

The market-clearing conditions in the money and rediscounting markets are

$$\bar{M}_t = M_t$$

and

$$\alpha \hat{n}_t = n_t,$$

respectively.

In what follows, we assume that the government pays a constant interest rate $1+i$ on reserve balances at all dates. Then, we refer to the interest rate $1+i$ as the policy rate. Using the previously derived first-order conditions, together with the market-clearing conditions, we obtain the following equilibrium relations:

$$x_t^y = e - m_t, \tag{8}$$

$$x_t^o = m_t + (1 - \rho_t)\,\hat{n}_t, \tag{9}$$

$$v'\left(x_t^y\right) = \frac{(1+i)\,\phi_{t+1}}{\phi_t \rho_{t+1}} \beta v'\left(x_{t+1}^o\right), \tag{10}$$

$$\left(\frac{1}{\rho_t} - 1\right)\left(\rho_t \hat{n}_t - m_t\right) = 0, \tag{11}$$

$$m_t = \frac{\phi_{t-1}}{\phi_t} m_{t-1}, \tag{12}$$

$$\frac{u'(q_t)}{w'(q_t)} = \frac{1}{\rho_{t+1}}, \tag{13}$$

$$w(q_t) = \alpha\beta\rho_{t+1}\hat{n}_{t+1}. \tag{14}$$

Condition (11) indicates whether the liquidity constraint is binding. Condition (13) gives DM output as a function of the discount rate.

Combining (13) and (14), we obtain per-capita debt issuance as a function of the discount rate, which is implicitly given by

$$\frac{u'\left(w^{-1}\left(\alpha\beta\rho_{t+1}\hat{n}_{t+1}\right)\right)}{w'\left(w^{-1}\left(\alpha\beta\rho_{t+1}\hat{n}_{t+1}\right)\right)} = \frac{1}{\rho_{t+1}}.$$

This is the familiar equilibrium relation that arises in the Lagos-Wright framework linking DM output and the rate of return on liquid assets. In our framework, the consumers do not hold assets in portfolio for transaction purposes. Instead, they issue personal debt claims that are rediscounted in the following period at the real price ρ_{t+1}, which in turn influences DM output in equilibrium.

We can now formally define an equilibrium in the intermediated economy as follows.

Definition 1 *An equilibrium can be defined as a sequence* $\{x_t^y, x_t^o, \hat{n}_t, m_t, \rho_t, \phi_t, q_t\}_{t=0}^{\infty}$ *satisfying (8)-(14) with* $\rho_t\hat{n}_t \leq m_t \leq e$ *at all dates.*

From now on, we restrict attention to stationary equilibria with the property that the value of reserves remains constant over time. Then, a stationary plan $(x^y, x^o, \hat{n}, m, \rho, q)$ must satisfy the following equilibrium conditions:

$$x^y = e - m, \tag{15}$$

$$x^o = m + (1-\rho)\hat{n}, \tag{16}$$

$$v'(x^y) = \frac{(1+i)}{\rho}\beta v'(x^o), \tag{17}$$

$$\left(\frac{1}{\rho} - 1\right)(\rho\hat{n} - m) = 0, \tag{18}$$

$$\frac{u'(q)}{w'(q)} = \frac{1}{\rho}, \tag{19}$$

$$w(q) = \alpha\beta\rho\hat{n}. \tag{20}$$

Because consumption is nonnegative, real balances must satisfy the upper bound $m \leq e$ in equilibrium. Additionally, the liquidity constraint imposes the lower bound $m \geq \rho\hat{n}$. Thus, in addition to the previously described conditions, a stationary equilibrium must satisfy the following boundary conditions:

$$\rho\hat{n} \leq m \leq e. \tag{21}$$

Thus, a stationary equilibrium can be defined as a plan $(x^y, x^o, \hat{n}, m, \rho, q)$ satisfying (15)-(21).

4.5. Existence and Properties

Let $\hat{\Psi}(\cdot)$ denote the inverse of $u'(\cdot)/w'(\cdot)$.[4] Then, equilibrium conditions (19) and (20) imply the following relation:

$$w\left(\hat{\Psi}\left(\frac{1}{\rho}\right)\right) = \alpha\beta\rho\hat{n}.$$

Define the function $\Psi(y) \equiv w\left(\hat{\Psi}(y)\right)$ for all $y > 0$. Then, per-capita rediscounting can be written as

$$\hat{n} = \frac{1}{\alpha\beta} \times \frac{1}{\rho}\Psi\left(\frac{1}{\rho}\right).$$

Given these equilibrium relations, we can rewrite the banker's Euler equation (17) as

$$v'(e-m) = \frac{(1+i)}{\rho}\beta v'\left(m + \frac{1}{\alpha\beta}\left(\frac{1-\rho}{\rho}\right)\Psi\left(\frac{1}{\rho}\right)\right). \tag{22}$$

Also, we can rewrite the complementary slackness condition (18) as

$$\left(\frac{1}{\rho} - 1\right)\left[\frac{1}{\alpha\beta}\Psi\left(\frac{1}{\rho}\right) - m\right] = 0. \tag{23}$$

Then, a steady state can be defined as a pair (m, ρ) satisfying equations (22) and (23) as well as the following inequalities:

$$\frac{1}{\alpha\beta}\Psi\left(\frac{1}{\rho}\right) \leq m \leq e.$$

Once we have determined real balances and the discount rate in a stationary equilibrium, we can derive the other equilibrium values by using (15)-(20).

[4] Recall that $u'(\cdot)/w'(\cdot)$ is strictly decreasing.

I. Nonbinding Liquidity Constraint

Consider now the existence of a steady state with a nonbinding liquidity constraint. As we have seen, a necessary condition for efficiency is to have a slack liquidity constraint so that debt claims trade at par value in the rediscounting market. In an equilibrium with a nonbinding liquidity constraint, we have $\rho = 1$. Then, equation (22) implicitly defines $m = m(i)$ as

$$v'(e - m(i)) = (1 + i)\beta v'(m(i)). \tag{24}$$

The other equilibrium quantities and prices are given by

$$\rho = 1, \tag{25}$$

$$q = q^*, \tag{26}$$

$$\hat{n} = \frac{w(q^*)}{\alpha\beta}, \tag{27}$$

$$x^y = e - m(i), \tag{28}$$

$$x^o = m(i). \tag{29}$$

Note that $x^y + x^o = e$ holds in equilibrium so that a banker earns zero profits when rediscounting debt claims. In that case, the banker's franchise value is zero, and the return on savings equals the return on assets (i.e., the banker earns no extra income from rediscounting). The amount of debt claims purchased by an old banker is given by (27).

To establish the existence of a steady state with a nonbinding liquidity constraint, we need to verify whether a slack liquidity constraint is consistent with the boundary equilibrium conditions. It turns out that a steady state with a nonbinding liquidity constraint exists provided real balances satisfy

$$\frac{w(q^*)}{\alpha\beta} \leq m(i). \tag{30}$$

Condition (30) states that the banker's real reserve holdings must be at least the same as the socially efficient per-capita discounting amount. Thus, an equilibrium with a nonbinding liquidity constraint requires a minimum income level in the second period of a banker's life cycle.

As we have seen, the banker's old-age income can be decomposed into two parts: (i) the returns from holding assets across periods and (ii) the earnings from rediscounting. Because no discounting is socially optimal, the banker's franchise value must be zero to obtain efficiency in the payment system. Then, it is necessary to make reserves an asset class that yields a high return across periods to attain the required old-age income consistent with no discounting. As we will show, depending on the parameters, this is not a sufficient condition for efficiency.

To clearly characterize the threshold for the policy rate $1+i$ satisfying (30), we make the following assumption throughout the paper.

Assumption 1: Assume that $v(x) = \ln x$.

Then, (24) implies

$$m(i) = \frac{e\beta(1+i)}{1+\beta(1+i)} \tag{31}$$

so that condition (30) becomes

$$1+i \geq \frac{1}{\beta\left[\frac{\alpha\beta e}{w(q^*)} - 1\right]}.$$

To obtain a well-defined demand function for real balances, the policy rate must satisfy the following condition:

$$1+i \leq \frac{1}{\beta}.$$

Thus, a necessary and sufficient condition for the existence of an equilibrium with a non-binding liquidity constraint is

$$\frac{\alpha\beta e}{2} \geq w(q^*). \tag{32}$$

If (32) holds, any policy rate $1+i$ satisfying

$$\frac{1}{\beta\left[\frac{\alpha\beta e}{w(q^*)} - 1\right]} \leq 1+i \leq \frac{1}{\beta} \tag{33}$$

leads to an equilibrium with a nonbinding liquidity constraint and with real balances given by (31). We summarize these results in the following proposition.

Proposition 2 *A stationary equilibrium with a nonbinding liquidity constraint exists if and only if* $\frac{\alpha\beta e}{2} \geq w(q^*)$ *and* $1+i$ *lies in the interval (33). The equilibrium quantities and prices satisfy (24)-(29).*

The previous proposition shows that an equilibrium without discounting exists provided the policy rate is sufficiently large under the parametric assumption $\frac{\alpha\beta e}{2} \geq w(q^*)$. As we have seen in the solution to the planner's problem, efficiency also requires perfect consumption smoothing for bankers. Given (24), we can attain perfect consumption smoothing for bankers by setting

$$1+i = \frac{1}{\beta}.$$

In other words, the central bank sets the interest rate on reserves at the upper bound. Such a policy prescription is a version of the Friedman rule, which eliminates the opportunity cost of holding money balances across periods for transaction purposes. At the Friedman rule, we have

$$m\left(\beta^{-1} - 1\right) = \frac{e}{2}$$

and

$$x^y = x^o = \frac{e}{2},$$

together with no discounting and the surplus-maximizing quantity q^* in the decentralized market. We summarize these results in the following proposition.

Proposition 3 *If* $\frac{\alpha\beta e}{2} \geq w(q^*)$*, then the Friedman rule* $1+i = \frac{1}{\beta}$ *leads to a socially efficient allocation.*

We can now provide some intuition for the condition $\frac{\alpha\beta e}{2} \geq w(q^*)$. It is helpful to rewrite it as

$$\frac{e}{2} \geq \frac{1}{\alpha} \times \frac{w(q^*)}{\beta}.$$

As previously mentioned, the right-hand side gives the per-capita discounting volume consistent with an efficient allocation. The left-hand side gives the initial per-capita wealth in the banking system at the Friedman rule. If the initial per-capita wealth in the banking system is at least as large as the socially efficient per-capita discounting volume, then the Friedman rule is consistent with an efficient allocation, and we say that there is abundant liquidity flowing into the rediscounting market to drive the discount rate to zero.[5]

II. Binding Liquidity Constraint

Now we consider the case $\frac{\alpha\beta e}{2} < w(q^*) \leq \alpha\beta e$. In this region of the parameter space, the liquidity constraint is binding for any value of the policy rate. Again, restricting attention to stationary equilibria, we obtain the following equilibrium relations:

$$x^y = e - \rho\hat{n},$$

$$x^o = \hat{n},$$

$$v'(x^y) = \frac{\beta(1+i)}{\rho} v'(x^o),$$

$$\frac{u'(q)}{w'(q)} = \frac{1}{\rho},$$

$$w(q) = \alpha\beta\rho\hat{n}.$$

Given any interest rate such that $1 + i \leq \beta^{-1}$, we can solve for the equilibrium quantities and prices to obtain the following allocation:

$$x^y = \frac{e}{1 + (1+i)\beta},$$

$$x^o = \hat{n} = \frac{\beta e}{\frac{1}{1+i} + \beta} \times \frac{u'\left(w^{-1}\left(\frac{\alpha\beta^2 e}{\frac{1}{1+i}+\beta}\right)\right)}{w'\left(w^{-1}\left(\frac{\alpha\beta^2 e}{\frac{1}{1+i}+\beta}\right)\right)},$$

$$\rho = \frac{w'\left(w^{-1}\left(\frac{\alpha\beta^2 e}{\frac{1}{1+i}+\beta}\right)\right)}{u'\left(w^{-1}\left(\frac{\alpha\beta^2 e}{\frac{1}{1+i}+\beta}\right)\right)} < 1,$$

[5] At the Friedman rule, the income of an old banker is $\frac{1}{\beta}\frac{e}{2}$. The government levies a lump-sum tax $-\left(1 - \frac{1}{\beta}\right)\frac{e}{2}$ to finance the implementation of the Friedman rule. Thus, the banker's available wealth in old age is $\frac{e}{2}$, which is the same as his initial wealth.

$$q = w^{-1}\left(\frac{\alpha\beta^2 e}{\frac{1}{1+i} + \beta}\right) < q^*.$$

As we can see, DM output is below the surplus-maximizing quantity for any value of the policy rate, and the banker's consumption profile is not perfectly smooth.

At the Friedman rule, we set the policy rate at the upper bound $1 + i \to \beta^{-1}$. The associated equilibrium allocation is given by

$$x^y = \frac{e}{2},$$

$$x^o = \hat{n} = \frac{e}{2}\frac{u'\left(w^{-1}\left(\frac{\alpha\beta e}{2}\right)\right)}{w'\left(w^{-1}\left(\frac{\alpha\beta e}{2}\right)\right)} > \frac{e}{2},$$

$$\rho = \frac{w'\left(w^{-1}\left(\frac{\alpha\beta e}{2}\right)\right)}{u'\left(w^{-1}\left(\frac{\alpha\beta e}{2}\right)\right)} < 1,$$

$$q = w^{-1}\left(\frac{\alpha\beta e}{2}\right) < q^*.$$

The liquidity constraint is binding (and the discount rate is strictly positive) even though the implementation of the Friedman rule has eliminated the opportunity cost of holding reserves across periods. To provide some intuition for this result, it is helpful to rewrite the condition $\frac{\alpha\beta e}{2} < w(q^*)$ as

$$\frac{e}{2} < \frac{1}{\alpha} \times \frac{w(q^*)}{\beta}.$$

As we can see, the socially efficient per-capita discounting volume is larger than the initial per-capita wealth in the banking system at the Friedman rule. As a result, there is an insufficient amount of resources flowing into the rediscounting market to purchase debt claims from merchants, resulting in a positive discount rate. The existence of discounting implies a strictly positive franchise value for the banker, which is socially inefficient. The following proposition summarizes these results.

Proposition 4 *If* $\frac{\alpha\beta e}{2} < w(q^*) \leq \alpha\beta e$*, the Friedman rule does not result in an efficient allocation.*

Our analysis shows that the Friedman rule is consistent with a socially efficient allocation only if the initial wealth in the banking system is sufficiently large relative to the size of the retail sector. Otherwise, there is no efficient equilibrium allocation. One reason for a relatively small amount of capital in the banking system is the existence of barriers to entry into banking and limits to the size of banks. If this situation arises, then there is insufficient capital in the intermediary sector to rediscount debt claims in consumer markets. As a result, the payment system is inefficient, which implies suboptimal production and consumption in the economy.

Note that the equilibrium discount rate is strictly decreasing in the policy rate $1+i$ when $\frac{\alpha\beta e}{2} < w(q^*) \leq \alpha\beta e$. Consequently, consumers are better off when the central bank raises the interest rate on reserves. The utility of bankers can either increase or decrease when the central bank raises the policy rate. For instance, if we assume that $u(q) = (1-\sigma)^{-1} q^{1-\sigma}$ with $0 < \sigma < 1$ and $w(q) = q$, then it can be shown that the indirect utility of bankers is strictly decreasing in $1+i$.[6] For this economy, the welfare effects of an increase in the policy rate are ambiguous.

So far we have considered an economy without interbank credit markets to investigate the extent to which monetary policy can enhance the functioning of the payment system even if interbank exchange is shut down completely. Our next step is to investigate whether the existence of interbank credit markets in the bankers' location makes the Friedman rule consistent with an efficient allocation regardless of the region in the parameter space.

5. INTERBANK MARKETS

We now consider the introduction of an interbank market in the bankers' location in which an old banker can borrow from young bankers an amount $b_t \in \mathbb{R}_+$ at the beginning of the period with repayment at the end of the period. Let $r_t \in \mathbb{R}_+$ denote the real interest rate on the loan. In what follows, we assume that the debt issued by bankers can be perfectly enforced in the bankers' location.

[6] Precisely, the slope of the indirect utility of bankers with respect to $1+i$ is $\frac{\beta}{1+\beta(1+i)}\left(\frac{1-\sigma}{1+i}-1\right) < 0$.

5.1. Bankers

Consider now the possibility of an interbank market in the bankers' location with repayment of debt claims at the end of the period. We can write the banker's problem as

$$\max_{\left(x_t^y, x_{t+1}^o, M_t, l_t, \hat{n}_{t+1}, b_{t+1}\right) \in \mathbb{R}_+^6} \left[v\left(x_t^y\right) + \beta v\left(x_{t+1}^o\right)\right]$$

subject to the first-period budget constraint

$$x_t^y + \phi_t M_t \leq e + r_t l_t,$$

the first-period liquidity constraint

$$\phi_t M_t + l_t \leq e,$$

the second-period budget constraint

$$x_{t+1}^o + \rho_{t+1} \hat{n}_{t+1} + r_{t+1} b_{t+1} \leq \phi_{t+1}\left(1+i\right) M_t + \hat{n}_{t+1} + \tau_{t+1},$$

and the second-period liquidity constraint

$$\rho_{t+1} \hat{n}_{t+1} \leq \phi_{t+1}\left(1+i\right) M_t + \tau_{t+1} + b_{t+1}.$$

Here, $l_t \in \mathbb{R}_+$ denotes the loan amount in the interbank market in period t, and $b_{t+1} \in \mathbb{R}_+$ denotes the amount borrowed in period $t+1$. Let $\mu_t \in \mathbb{R}_+$ denote the Lagrange multiplier on the first-period liquidity constraint, and let $\beta \lambda_{t+1} \in \mathbb{R}_+$ denote the Lagrange multiplier on the second-period liquidity constraint. The first-order conditions are

$$r_t v'\left(x_t^y\right) - \mu_t = 0,$$

$$-\phi_t v'\left(x_t^y\right) - \mu_t + \beta\left(1+i\right)\phi_{t+1} v'\left(x_{t+1}^o\right) + \lambda_{t+1} \beta \phi_{t+1} = 0,$$

$$\left(1-\rho_{t+1}\right) v'\left(x_{t+1}^o\right) - \lambda_{t+1} \rho_{t+1} = 0,$$

$$-v'\left(x_{t+1}^o\right) r_{t+1} + \lambda_{t+1} = 0,$$

$$x_t^y = e + r_t l_t - \phi_t M_t,$$

$$x_{t+1}^{o}=\phi_{t+1}\left(1+i\right)M_t+\tau_{t+1}+\left(1-\rho_{t+1}\right)\hat{n}_{t+1}-b_{t+1}r_{t+1}.$$

In addition, we have the complementary slackness conditions:

$$\lambda_{t+1}\left[\rho_{t+1}\hat{n}_{t+1}-\phi_{t+1}\left(1+i\right)M_t-\tau_{t+1}-b_{t+1}\right]=0$$

and

$$\mu_t\left(\phi_t M_t+l_t-e\right)=0.$$

Following the same steps as in the previous section, we can use the first-order conditions to derive the following equilibrium relations:

$$v'\left(x_t^{y}\right)=\beta\frac{\phi_{t+1}}{\left(\phi_t+r_t\right)\rho_{t+1}}v'\left(x_{t+1}^{o}\right)$$

and

$$r_{t+1}=\frac{1}{\rho_{t+1}}-1.$$

The first relation is the Euler equation for the intertemporal allocation of consumption. The effective real return on assets is now $\frac{\phi_{t+1}}{(\phi_t+r_t)\rho_{t+1}}$, given that bankers can borrow in the interbank credit market to rediscount debt claims in the central location.

The second relation is a no-arbitrage condition. In other words, there are no arbitrage opportunities in the interbank market provided that the real return on each unit of debt rediscounted in the central location equals the cost of borrowing in interbank markets.

5.2. Equilibrium

To construct an equilibrium allocation, we use the market-clearing conditions in the money and rediscounting markets, as in the previous section, and add the market-clearing condition in the interbank market:

$$l_t=b_t.$$

Then, we can combine the first-order conditions for the banker's problem and for the consumer's problem with the market-clearing conditions to derive the following equilibrium relations:

$$x_t^{y}=e+r_t l_t-m_t, \tag{34}$$

$$x_t^o = m_t + (1 - \rho_t)\,\hat{n}_t - l_t r_t, \tag{35}$$

$$v'\left(x_t^y\right) = \frac{(1+i)\,\phi_{t+1}}{(\phi_t + r_t)\,\rho_{t+1}} \beta v'\left(x_{t+1}^o\right), \tag{36}$$

$$\left(\frac{1}{\rho_t} - 1\right)(\rho_t \hat{n}_t - m_t - l_t) = 0, \tag{37}$$

$$r_{t+1} = \frac{1}{\rho_{t+1}} - 1, \tag{38}$$

$$r_t\,(m_t + l_t - e) = 0, \tag{39}$$

$$m_t = \frac{\phi_{t-1}}{\phi_t} m_{t-1}, \tag{40}$$

$$\frac{u'(q_t)}{w'(q_t)} = \frac{1}{\rho_{t+1}}, \tag{41}$$

$$w\,(q_t) = \alpha\beta\rho_{t+1}\hat{n}_{t+1}. \tag{42}$$

In addition, an equilibrium allocation must satisfy the following boundary conditions:

$$\rho_t \hat{n}_t \leq m_t + l_t \leq e \tag{43}$$

at all dates. Condition (38) indicates that a positive interest rate in the interbank market necessarily implies a binding liquidity constraint in the rediscounting market. A positive interest rate also implies a binding liquidity constraint for old bankers when borrowing in the interbank market.

We can now formally define an equilibrium for the economy with complete interbank markets.

Definition 5 *An equilibrium can be defined as a sequence* $\{x_t^y, x_t^o, \hat{n}_t, m_t, l_t, \rho_t, \phi_t, q_t\}_{t=0}^{\infty}$ *satisfying (34)-(43) at all dates.*

As in the previous section, we restrict attention to equilibria in which the value of reserves is constant over time.

5.3. Existence and Properties

In what follows, we focus on stationary allocations in which the discount rate and the interest rate remain constant over time. A stationary equilibrium for the economy with interbank markets can be defined as a plan $(x^y, x^o, \hat{n}, m, l, \rho, \phi, q)$ satisfying

$$x^y = e + rl - m,$$

$$x^o = m + (1-\rho)\,\hat{n} - lr,$$

$$v'(x^y) = \frac{(1+i)\,\phi}{(\phi + r)\,\rho}\beta v'(x^o),$$

$$\left(\frac{1}{\rho} - 1\right)(\rho\hat{n} - m - l) = 0,$$

$$r = \frac{1}{\rho} - 1,$$

$$r\,(m + l - e) = 0,$$

$$\frac{u'(q)}{w'(q)} = \frac{1}{\rho},$$

$$w(q) = \alpha\beta\rho\hat{n},$$

together with the boundary conditions $\rho\hat{n} \leq m + l \leq e$.

Consider now the existence of a stationary equilibrium with a nonbinding liquidity constraint. Note that $\rho = 1$ implies $r = 0$ so that a stationary equilibrium with a nonbinding liquidity constraint must satisfy

$$x^y = e - m,$$

$$x^o = m,$$

$$v'(x^y) = (1+i)\,\beta v'(x^o),$$

$$q = q^*,$$

$$\hat{n} = \frac{w(q)}{\alpha\beta},$$

together with the boundary conditions

$$\hat{n} \leq m + l \leq e.$$

A nonbinding liquidity constraint requires

$$\frac{w(q^*)}{\alpha\beta} \le m(i) + l,$$

where $m(i)$ is defined as in (24). The main difference from the equilibrium conditions when interbank credit was not possible is the presence of the loan amount l on the right-hand side of the previous condition. Given these restrictions on the equilibrium allocation, we can define the equilibrium loan amount as

$$l = l(i) \equiv \max\left\{\frac{w(q^*)}{\alpha\beta} - m(i), 0\right\}$$

so that the liquidity constraint holds as an equality. If we choose the loan amount in this way, then we have

$$l(i) + m(i) = \frac{w(q^*)}{\alpha\beta} \le e$$

for any value of the policy rate in the interval $0 \le 1 + i \le \beta^{-1}$. Because the interest rate is zero, the loan amount $l(i)$ is consistent with market clearing for any $0 \le 1 + i \le \beta^{-1}$. Thus, it is possible to construct an equilibrium with a nonbinding liquidity constraint in the economy with interbank markets regardless of the level of the policy rate. We summarize these findings in the following proposition.

Proposition 6 *In the economy with interbank markets, an equilibrium with a nonbinding liquidity constraint exists for any value of the policy rate.*

Under complete interbank markets, there is an equilibrium without discounting for any choice of the policy rate. Allowing bankers to trade in a frictionless credit market with perfect enforcement of debt claims is sufficient to overcome the payment frictions in the model (i.e., spatial separation and unsynchronized trading patterns). However, the allocation is not always efficient because the policy rate may not imply a smooth consumption profile for bankers. Thus, our next step is to show that the Friedman rule is consistent with an efficient allocation in the economy with complete interbank markets by allowing a smooth intertemporal exchange.

Assumption 1 implies

$$l(i) = \max\left\{\frac{w(q^*)}{\alpha\beta} - \frac{e\beta(1+i)}{1+\beta(1+i)}, 0\right\}$$

so that, at the Friedman rule, we have

$$l\left(\beta^{-1} - 1\right) = \frac{w(q^*)}{\alpha\beta} - \frac{e}{2} \leq \frac{e}{2}.$$

As in the previous section, it follows that $x^y = x^o = \frac{e}{2}$, so we attain perfect consumption smoothing for bankers. We summarize these results in the following proposition.

Proposition 7 *The Friedman rule is consistent with an efficient allocation in the economy with interbank markets.*

In the absence of interbank markets, the Friedman rule can deliver an efficient allocation only if $\frac{\alpha\beta e}{2} \geq w(q^*)$. Otherwise, the discount rate is positive and the allocation is inefficient. The introduction of complete interbank markets allows old bankers to borrow from young bankers to increase their purchases in the rediscounting market, so the payment frictions are not binding. The Friedman rule ensures that bankers do not economize on reserves when transferring wealth across periods so that their consumption profile is perfectly smooth, as in the solution to the planner's problem.

Note that we can achieve the same outcome in the economy with complete markets if we add a liquidity facility to the incomplete markets economy described in Section 4. Specifically, suppose that we have a government-operated discount window that takes deposits from young bankers and makes loans to old bankers. When the interest rate on deposits at the liquidity facility is the same as the lending rate, the budget set for the banker is exactly the same as that described in Section 5.1. As a result, the set of equilibrium allocations in the incomplete markets economy with a liquidity facility is the same as that of the complete markets economy.

6. DISCUSSION

The main result in this paper is to show that monetary policy plays an important role in the efficient operation of the payment system when interbank credit markets are incomplete.

As we have seen, it is possible to construct an equilibrium without discounting when the initial wealth in the banking system is sufficiently large relative to the size of the retail sector, provided the nominal interest rate on reserves is sufficiently close to the rate of time preference. Note that we considered an extreme situation in which interbank credit markets were completely shut down. In that case, the central bank can overcome the payment frictions in the model by making reserves an asset class that yields a sufficiently large return to guarantee the smooth functioning of the payment system.

If the fundamentals of the economy are such that an equilibrium without discounting does not exist, then monetary policy can increase the welfare of consumers by raising the interest rate on reserves, which results in a lower discount rate in equilibrium. However, the welfare of intermediaries is likely to decline with an increase in the discount rate. As we have seen, the indirect utility of intermediaries declines with an increase in the interest rate on reserves under standard assumptions for preferences and technologies. Specifically, a change in the interest rate on reserves that leads to a decrease in the discount rate results in a reduction in the intermediary's franchise value that is not offset by an increase in the yield on reserve holdings, resulting in a lower level of utility for intermediaries. Thus, the welfare effects of monetary policy are ambiguous when the fundamentals of the economy are such that the initial wealth in the banking system is small relative to the size of the retail sector.

One reason for a relatively small amount of capital in the banking system is the existence of barriers to entry into the banking sector and limits to the size of banks. If this situation arises, then there is insufficient capital in the intermediary sector to rediscount debt claims in consumer markets, resulting in an inefficient payment system and in suboptimal production and consumption in decentralized markets.

Our analysis suggests that promoting entry into the rediscounting business can lead to an efficient payment system. As we have seen, the Friedman rule can deliver an efficient allocation only if $\frac{\alpha\beta e}{2} \geq w(q^*)$. In our framework, an increase in the number of bankers operating in the rediscounting market would lead to a larger value for α. Consequently, it would make it more likely that $\frac{\alpha\beta e}{2} \geq w(q^*)$ so that setting the interest rate on reserves equal to the rate of time preference would lead to an efficient allocation.

The Friedman rule is the optimal monetary policy across all market structures considered in this paper, so our analysis confirms the view that efficiency in the payment system and in the allocation of resources in other sectors of the economy requires setting the interest rate on reserves equal to the rate of time preference. By eliminating the opportunity cost of holding reserves across periods, the central bank promotes a smoother consumption profile for intermediaries. Depending on the fundamentals of the economy, the implementation of the Friedman rule can replicate the solution to the planner's problem by perfectly smoothing the consumption profile of intermediaries.

As we have mentioned in the paper, our framework provides a useful model with payment and monetary frictions that can be easily compared with other papers in the New Monetarist literature, as opposed to the original Freeman (1996a) model which builds on a specific class of overlapping generation models and requires special assumptions regarding production possibilities. The bankers in our framework have a fixed endowment of goods and their actions as financial intermediaries influence DM and CM output via the rediscounting market, which is a crucial part of the economy's credit system. Thus, aggregate output and welfare in our framework can be easily compared to that in other papers in the literature using the Lagos-Wright framework.

The extent to which intermediation has real effects in our economy depends on the degree of unsynchronicity that is added to the model. In this paper, we have considered a version of the model in which consumers and merchants do not overlap *at all* in the central location. It is, however, possible to consider a version of the model in which a fraction of consumers and merchants do overlap in the central location. In this version of the model, the real effects of intermediation are smaller as the payment friction is less severe.

On the other hand, if we completely removed the assumption of unsynchronized trading patterns in the economy (i.e., if we assumed that consumers and merchants do overlap in the central location), then our model would collapse to a standard Lagos-Wright model with perfect credit, as that described in Nosal and Rocheteau (2017). As we have seen, the economy with a frictionless credit system can be replicated as an equilibrium outcome of the economy with payment frictions, depending on the parameters of the model, if the central bank provides the right incentives to intermediaries by setting the interest rate on reserves equal to the rate of time preference.

7. CONCLUSIONS

We have constructed a model of consumer credit with payment frictions, such as spatial separation and unsynchronized trading patterns, to study optimal monetary policy across different interbank market structures. In our framework, intermediaries play an essential role in the functioning of the payment system by rediscounting debt claims originated in retail transactions. The timing of events within the period is such that intermediaries must hold reserves to transfer wealth across periods to take advantage of rediscounting opportunities. Monetary policy influences the equilibrium allocation through the interest rate on reserves.

We have shown that, in the absence of complete interbank credit markets, monetary policy is an essential tool to guarantee the efficient functioning of the payment system. Monetary policy can lead to an equilibrium in which consumers' debt claims are not discounted so that exchange through the intermediated credit system is efficient. Monetary policy cannot always implement an efficient allocation when interbank markets are incomplete, but it can enhance the functioning of the payment system by lowering the discount rate. As we have seen, the welfare effects of monetary policy can be ambiguous, given that a lower equilibrium discount rate results in a smaller franchise value for intermediaries involved in rediscounting consumers' debt claims. Finally, we have argued that promoting entry into the rediscounting business can lead to an efficient payment system by driving the discount rate to zero, holding other factors constant.

REFERENCES

[1] S. Freeman. "The payments system, liquidity, and rediscounting," *American Economic Review* 86 (1996a), pp. 1126-1138.

[2] S. Freeman. "Clearinghouse banks and banknote over-issue," *Journal of Monetary Economics* 38 (1996b), pp. 101-115.

[3] M. Friedman. *A Program for Monetary Stability*. New York: Fordham University Press (1959).

[4] M. Friedman. "The optimum quantity of money" in *The Optimum Quantity of Money and Other Essays*. Chicago: Aldine Publishing Company (1969).

[5] P. Gomis-Porqueras, D. Sanches. "Optimal monetary policy in a model of money and credit," *Journal of Money, Credit and Banking* 45 (2013), pp. 701-730.

[6] C. Kahn, W. Roberds. "Payment system settlement and bank incentives," *Review of Financial Studies* 11 (1998), pp. 845-870.

[7] C. Kahn, W. Roberds. "Why pay? An introduction to payments economics," *Journal of Financial Intermediation* 18 (2009), pp. 1-23.

[8] R. Lagos, R. Wright. "A unified framework for monetary theory and policy analysis," *Journal of Political Economy* 113 (2005), pp. 463-484.

[9] A. Martin. "Optimal pricing of intraday liquidity," *Journal of Monetary Economics* 51 (2004), pp. 401-424.

[10] D. Mills. "Alternative central bank credit policies for liquidity provision in a model of payments," *Journal of Monetary Economics* 53 (2006), pp. 1593-1611.

[11] C. Monnet, W. Roberds. "Optimal pricing of payment services," *Journal of Monetary Economics* 55 (2008), pp. 1428-1440.

[12] E. Nosal, G. Rocheteau. *Money, Payments, and Liquidity*, Second edition. Cambridge, MA: MIT Press (2017).

[13] G. Rocheteau, R. Wright. "Money in search equilibrium, in competitive equilibrium, and in competitive search equilibrium," *Econometrica,* 73 (2005), pp. 175-202.

[14] D. Sanches, S. Williamson. "Money and credit with limited commitment and theft," *Journal of Economic Theory* 145 (2010), pp. 1525-1549.

[15] T. Temzelides, S. Williamson. "Payments system design in deterministic and private information environments," *Journal of Economic Theory* 99 (2001), pp. 297-326.

[16] S. Williamson. "Payments systems with random matching and private information," *Journal of Money, Credit and Banking* 30 (1998), pp. 551-569.

[17] S. Williamson, R. Wright. "New monetarist economics: Methods," *Federal Reserve Bank of St. Louis Review* 92 (2010a), pp. 265-302.

[18] S. Williamson, R. Wright. "New monetarist economics: Models," *Handbook of Monetary Economics*, (eds.) B. Friedman and M. Woodford. Amsterdam: Elsevier (2010b).

[19] M. Woodford. "The optimum quantity of money," *Handbook of Monetary Economics*, vol. 2, (eds.) B. Friedman and F.H. Hahn. Amsterdam: North-Holland (1990).