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Liquidity, Trends, and the Great Recession

Pablo A. Guerron-Quintana Ryo Jinnai*

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Abstract

We study the impact that the liquidity crunch in 2008-2009 had on the U.S. economy's growth trend. To this end, we propose a model featuring endogenous productivity á la Romer and a liquidity friction á la Kiyotaki-Moore. A key finding in our study is that liquidity declined around the Lehman Brothers' demise, which led to the severe contraction in the economy. This liquidity shock was a tail event. Improving conditions in financial markets were crucial in the subsequent recovery. Had conditions remained at their worst level in 2008, output would have been 20 percent below its actual level in 2011. We show that a subsidy to entrepreneurs would have gone a long way averting the crisis.

1 Introduction

A few years into the recovery from the Great Recession, it is becoming clear that real GDP is failing to recover. Namely, the economy is growing at pre-crisis growth rates, but the crisis seems to have impinged a shift upon output. Figure 1 shows real GDP and its growth rate over the past decade. Without much effort, one can see that the economy is moving along a (new) trend that lies below the one prevailing in 2007.[1] It is also apparent that if the economy continues to display the dismal post-crisis growth rates (blue dashed line), it will not revert to the old trend.[2] Hence,

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[1] More formally, the shift in the GDP trend is detected by the flexible estimation of trends with regime shifts recently advanced by Eo and Kim (2012). We thank Yunjong Eo for helping with the estimation using their approach.

[2] The forecast is built assuming that the economy will be growing at the average growth rate for the period 2009.Q2 - 2013.Q2.

this tepid recovery has spurred debate about whether the shift is permanent and, if so, what the long-term implications are for the economy.[3] In this paper, we tackle the issue of the long-term impact of the Great Recession by means of a structural model.

An emerging consensus among economic observers is that, to some degree, the Great Recession was exacerbated by a financial shock (Brunnermeier, Eisenbach, and Sannikov (2012); Christiano, Motto, and Rostagno (2014); and Stock and Watson (2012)). More precisely, the liquidity crunch following the collapse of Lehman Brothers in 2008 has often been blamed for the depth and length of the recession and the subsequent sluggish recovery. Yet, formalizing this view in RBC-style models demands stringent conditions. First of all, shocks in this class of models exhibit exclusively short-run dynamics; i.e., the economy always reverts back to its pre-shock trend. One possibility is to assume a permanent shift in financial conditions, which presumably has a permanent impact on the resource allocation. A derailment of the U.S. economy from a linear trend may be seen as an adjustment toward the new steady state in which the economy produces fewer goods and services at the same technology level. But as we show in the next section, different financial indicators, such as liquidity, spreads, and lending activity, have recovered since the end of the crisis, and hence the evidence is hard to square with this hypothesis.[4]

A second and rather mechanical fix to this conundrum is assuming a break in productivity around the crisis. Such a shock can in principle "explain" a permanent shift in the trend line. The liquidity crunch might contribute to the severity of the recession, but it has nothing to do with the trend. While plausible as an explanation, we find this approach inflexible because it excludes the possibility that the liquidity crunch is a cause of the permanent shift in the trend. These considerations lead us to construct an alternative, more flexible model in which all the structural shocks have potential to influence the trend.

The model is based on the framework of Romer (1990). In the model, investment in research and development leads to the creation of new intermediate goods. A final goods producer takes these inputs to manufacture goods that are consumed and used for investment. Knowledge spillover sustains growth in the long run. The second key element in our model is a financial friction. Here, we follow the lead of Kiyotaki and Moore (2012) in assuming that financial frictions alter the liquidity of equity in the economy. More pointedly, shocks arising in the financial sector affect the resaleability of assets. In their formulation, a drop in liquidity reduces the availability of funds to finance new projects, leading to contraction in investment. In our model, this lack of funding leads to a low level of innovative activities, to weak knowledge spillover, and, hence, to (other

[3] This debate has received prominent attention in economic blogs, like those maintained by John Cochrane, John Taylor, and Stephen Williamson. A more provocative argument that declares the end of growth in the U.S. has been advanced in Gordon (Why Innovation Won't Save Us. The Wall Street Journal, December 21, 2012).

[4] Other economic factors might cause a permanent change in the resource allocation. Among many possibilities, we find changes in labor market conditions most interesting. See Nakajima (2012), for example. Our focus on the R&D and financing channels is not exclusionary to the analysis focusing on the labor market. We view our study as complementary to this interesting literature.

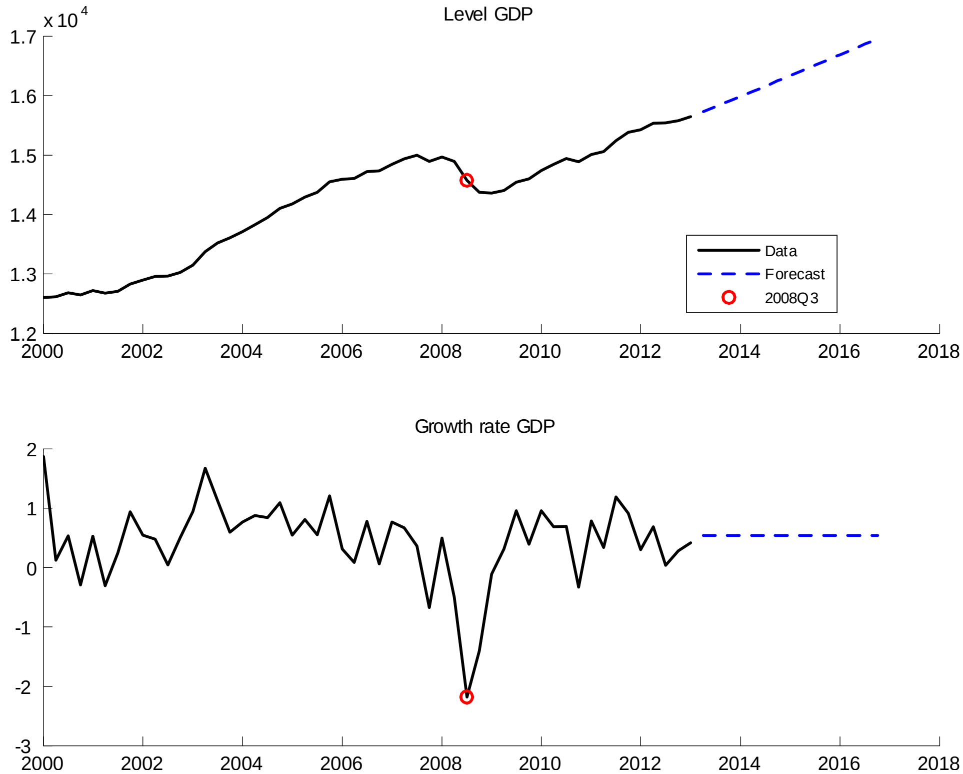

Figure 1: U.S. Real GDP

things being equal) a permanent shift in the economy's trend.

With our proposed model in hand, we read the recent history of the U.S. economy. Specifically, we use data on economic activity including a measure of research and development to estimate the stochastic properties of the structural shocks in the model. The results from the estimation exercise provide a vivid description of the events before, during, and after the Great Recession. Chief among these findings is that our measure of liquidity reached its lowest level just after Lehman Brothers' demise. Interestingly, this measure reaches its highest value (meaning that assets are most liquid) around the same time as the peak of the credit boom estimated by Ivashina and Scharfstein (2010).[5] By relying on simulations of alternative recovery paths, we uncover that improvements in financial markets (measured by the degree of market liquidity) were critical in pushing the economy out of the recession. These results nicely square with Stock and Watson (2012)'s view that financial shocks were one of the key drivers of the recession (the other one being uncertainty shocks). It is assuring that their study and ours arrive at similar conclusions from different paths. That is, Stock and Watson rely on a dynamic factor model whereas we dissect the data using a structural macroeconometric approach. Interestingly, our liquidity measure closely tracks their measure of financial distress during the crisis. Indeed, the correlation between the two variables is about 0.82, which we consider as favorable external evidence supporting our approach.

[5] They report that syndicated loans to corporations reached its highest value in billions of U.S. dollars in the second quarter of 2007. This is also true of the total number of loans.

We also read the U.S. data through the lens of a standard RBC model augmented with our financial friction and exogenous non-stationary productivity. Three main messages emerge from the model. First, the estimated liquidity process points to favorable financial conditions around Lehman Brothers' demise. It is only by the second half of 2009 that liquidity became adverse. These accounts, however, are contradictory to the micro evidence we show in the next section as well as to anecdotal reports of the crisis. Second, these adverse conditions have little impact on the economy. But this implication is not in accordance with the emerging consensus that the financial shock played a vital role during the Great Recession. Finally, the productivity shock is critical. This is hardly a surprising finding of the RBC model because it is (by construction) the only shock that can create the permanent shift in the trend from 2009 and beyond. But what seems to be counterfactual is the concentration of large negative productivity shocks; in fact, we identify, by means of simulations, an unusually large negative productivity shock in 2008.Q3 as the single dominant factor that explains the Great Recession. In reality, none of the off-the-shelf candidate productivity shocks (commodity prices, natural disasters, weather, etc.) displays a discontinuity in that quarter to the extent that it permanently changes the trend that much.

We also provide fresh evidence on the severity of the crisis, but from the perspective of an endogenous growth model. We find that the size of the financial shock around the Lehman episode was a tail event. Namely, based on the history of liquidity shocks, the Lehman shock was an event of probability of less than 0.1 percent.

We would like to stress that although the focal point of our discussion is the role of liquidity during and after the crisis, it does not mean that other aspects of the financial crisis (such as mortgage defaults and idiosyncratic risk at the firm level) were unimportant. On the contrary, the crisis was a multidimensional problem of which liquidity was one of the key elements. In this respect, we view our work as complementary to the studies focusing on the other aspects of the financial crisis. By the same token, our use of the Romer's endogenous growth model does not mean that our results crucially depend on this model's unique structure. We use this model primarily because it is parsimonious. Results similar to those discussed next should follow from other, possibly more elaborated versions of endogenous growth models too.

Our paper relates to several branches in macroeconomics. The first one comes from the literature on endogenous growth with seminal contributions by Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1997). Our analysis of the recent financial crisis brings us close to the literature on financial frictions in dynamic stochastic setups such as Bernanke, Gertler, and Gilchrist (1998), Jermann and Quadrini (2012), Kiyotaki and Moore (1997), and, more recently, Ajello (2012), Del Negro, Eggertsson, Ferrero, and Kiyotaki (2011), and Kiyotaki and Moore (2012). The empirical treatment used in our paper relates to the extensive literature on the estimation of dynamic stochastic general equilibrium models (Fernández-Villaverde, Guerrón-Quintana, and Rubio-Ramírez (2010) and Guerrón-Quintana (2010)). Finally, we borrow ideas from the unified treatment of business cycles and long-term dynamics in Comin and Gertler (2006), Comin, Gertler, and Santacreu (2008), Kung and Schmid (Forthcoming), and Queralto (2013).

The rest of the paper is organized as follows. The next section provides some financial indicators before, during, and after the 2008/2009 recession. Section 3 outlines the model and discusses equilibrium conditions. Our empirical strategy as well as the main results from our model are discussed in Section 4. Some discussion of our model-implied measure of liquidity is in Section 5. The last section provides some concluding remarks.

2 Some evidence on the liquidity crunch

A measure of liquidity usually used in the finance literature comes from margins for S&P 500 futures (Brunnermeier and Pedersen (2009)). The higher the margin is, the larger the amount of money an investor must maintain in a future contract. According to these authors, margins tend to increase during periods of liquidity crises. Indeed, they show that margins moved up in previous periods of illiquidity, as in 1987 (Black Monday) or 1998 (Asian and LTCM crises). Figure 2 shows these margins for the last decade.[6] As one can see, the most recent crisis led to a spike in the margins. At the peak in 2009, financiers required investors to keep 12 percent of the value of a future contract as a capital requirement. Note, however, that this measure of liquidity indicates that financial conditions started to improve by 2011, and seem to be back to more normal levels by the end of 2012. As we will see later, our estimated measure of liquidity displays remarkably similar dynamics, with the worst of the crisis happening in late 2008 and early 2009.

Figure 3 displays results from the survey of senior loan officers on bank lending practices published by the Federal Reserve Board. The survey dates back to 1990, so it is suitable to use for a comparison between the recent crisis and those in 1990/1991 and 2001.[7] The upper panel plots the net percentage of responders who answered that standards for commercial and industrial loans have tightened over the past three months in their banks. According to the survey, about 80 percent of loan officers reported tighter lending standards in the aftermath of Lehman's collapse. Small and large firms seem to have been affected equally by more stringent financing conditions. None of the previous two recessions saw a similar spike in this measure of tougher lending standards. The second panel in Figure 3 displays the time path of the net percentage of responders reporting an increasing gap between loan rates and the bank's cost of funding. The spike in spreads in 2008 shows that businesses (commercial and industrial; large and small) faced adverse financing conditions during the last recession. In the appendix, we provide

[6] The margins are computed as the dollar margin divided by the product of the underlying S&P 500 index and the size of the contract ($250 in this case). Data for margins are taken from Chicago Mercantile Exchange's website (http://www.cmegroup.com/clearing/risk-management/historical-margins.html). We thank Ronel Elul for helping with computation.

[7] The survey asks senior loan officers about "changes in the standards and terms on bank loans to businesses and households over the past three months." The most recent survey at the time of this writing (July 2013) included responses from officers at 73 domestic banks and 22 U.S. branches and agencies of foreign banking institutions.

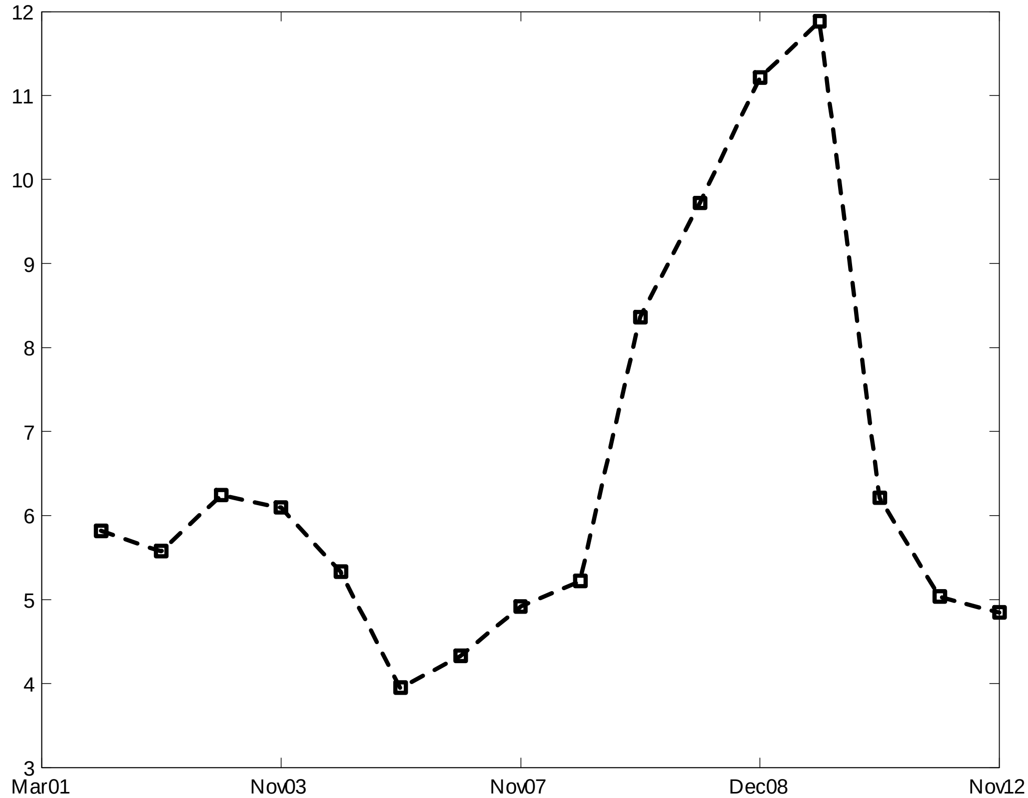

Figure 2: Margins for S&P 500 Futures

additional evidence on the severity of the financial crunch based on credit spreads.

Ivashina and Scharfstein (2010) analyze syndicated loans, the main vehicle through which banks lend to large corporations. This market is a part of the "shadow banking" system because non-bank financial institutions are often involved in sharing a loan originated by a lead bank. They report that the total volume of new syndicated loans fell by 47 percent during the peak period of the financial crisis (fourth quarter of 2008) relative to the prior quarter, and by 79 percent relative to the peak of the credit boom (second quarter of 2007). While commercial and industrial loans reported on the aggregate balance sheet of the U.S. banking sector sharply rose in the four weeks after the failure of Lehman Brothers (Chari, Christiano, and Kehoe (2008)), Ivashina and Scharfstein argue that this increase was actually consistent with the decline in syndicated lending because it was driven by an increase in drawdowns by corporate borrowers on existing credit lines (i.e., prior commitments by banks to lend to corporations at pre-specified rates and up to pre-specified limits).

On the cause of the dramatic shrinkage in lending activities, a number of studies report evidence suggesting that it was largely driven by an exogenous reduction in credit. Almeida, Campello, Laranjeira, and Weisbenner (2009) compare firms that needed to refinance a substantial fraction of their long-term debt over the year following August 2007 with firms that do not have a large refinancing in the period following the start of the financial crisis. After controlling other firm

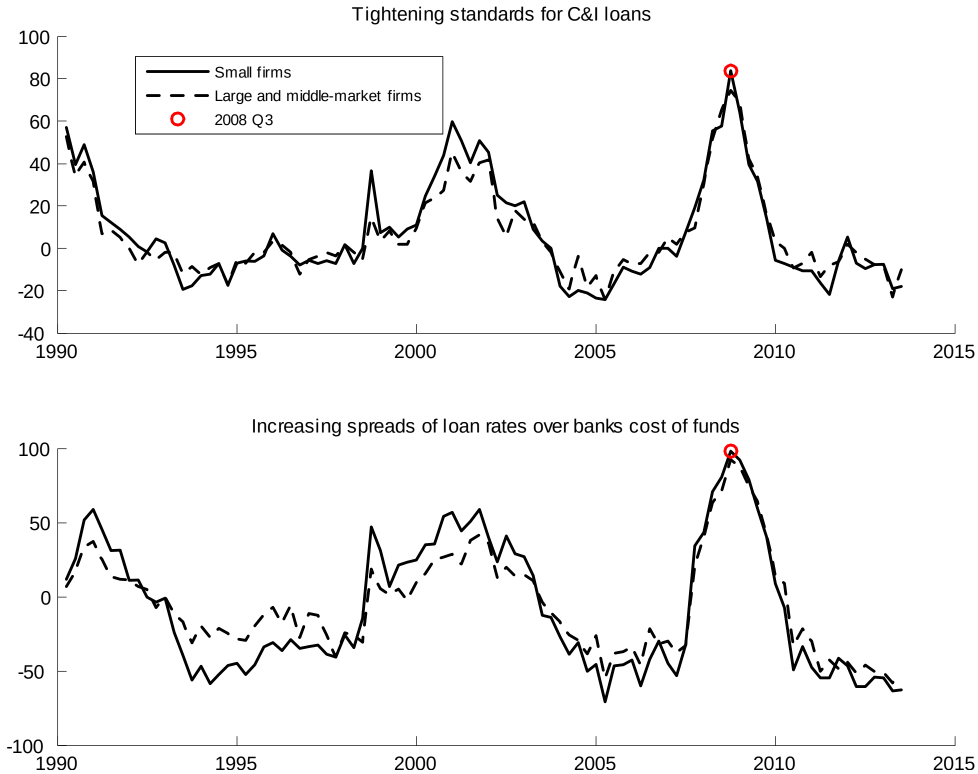

Figure 3: Senior Loan Officer Opinion Survey on Bank Lending Practices

characteristics using a matching estimator, they find that investment of firms in the first group fell by one-third, while investment in the second group showed no investment reduction. Duchin, Ozbas, and Sensoy (2010) find a similar result by comparing firms that were carrying more cash prior to the onset of the crisis with firms that were carrying less cash. Campello, Graham, and Harvey (2010) surveyed 1,050 chief financial officers (CFOs) in 39 countries in the middle of the crisis and found that, after controlling other firm characteristics using a matching estimator, financially constrained firms planned to cut more investment, technology, marketing, and employment relative to financially unconstrained firms; to restrict their pursuit of attractive projects; and to cancel valuable investments. Interestingly, they also found that financially constrained firms accelerate the withdrawal of funds from their outstanding line of credit, which is consistent with Ivashina and Scharfstein (2010).

Our final evidence on the liquidity crunch comes from private equity investment data. Figure 4 plots total private equity investment as well as some of its components expressed as fractions of GDP.[8] To the extent that startups and entrepreneurs rely on private funding to finance their operations, the collapse of private equity investment (either total or its components) in 2009 indicates that otherwise profitable projects may have had a hard time securing financing during the Great Recession. In other words, the financial headwinds in 2008/2009 effectively reduced the

[8] A plot in levels (rather than ratios) reveals a similar contraction in 2008/2009. The data are retrieved from Thomson One Analytics.

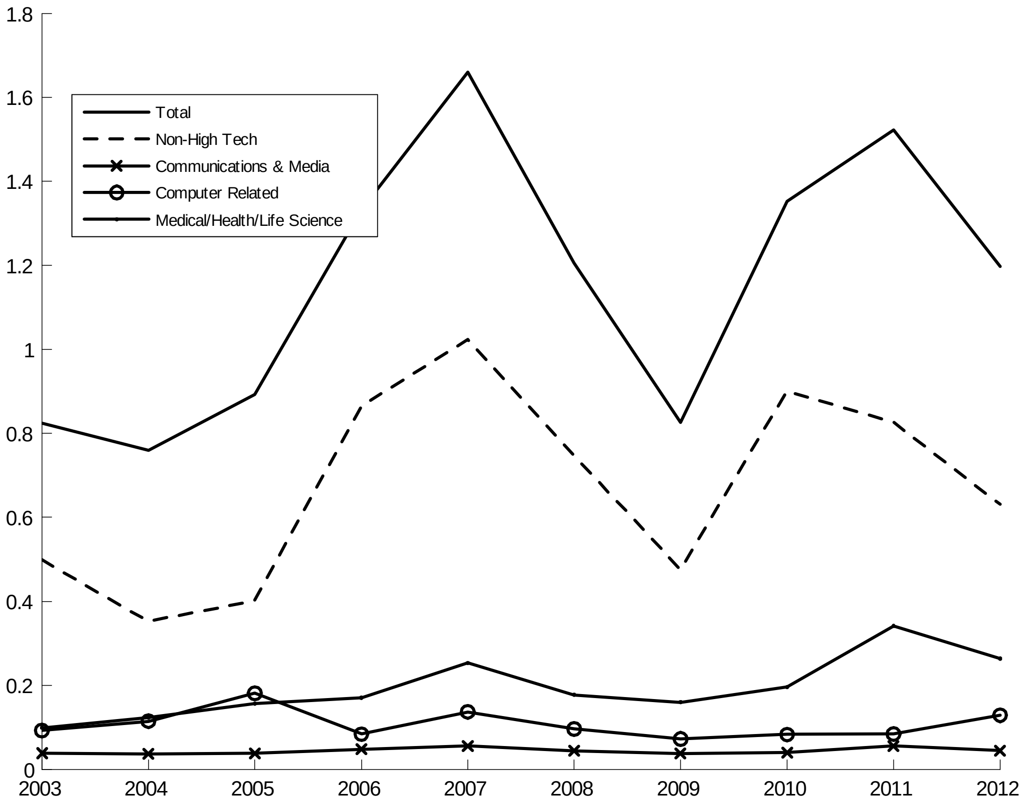

Figure 4: Private Equity Investment as a Fraction of GDP

liquidity of equity.[9] In the next section, we develop a model that incorporates changes in liquidity and allows for these fluctuations to affect the growth path of the economy. Related to this point, the U.S. Patent and Trademark Office reports a slowdown in the growth of the number of utility patents awarded during the Great Recession.[10] The growth rate has strongly recovered since 2010, increasing by more than 1 percentage point. Although there are a number of reasons why the number of awarded patents varies (including changes in the speed of the reviewing process), we find it very suggestive that this measure with a clear connection to our formulation hit a bump during the recession.

3 Model

We describe our baseline model in two steps. First, we flesh out the household side, where the financial friction takes place. Then we switch to the endogenous growth part of the model, which

[9] There is a plethora of anecdotal evidence on the liquidity crunch. For example, in a recent *Wall Street Journal* article (10/06/2013), it is reported that Berkshire Hathaway invested up to $25 billion during the crisis (when credit markets were tight) in big corporations needing funding, such as Mars, Goldman Sachs, General Electric, and Dow Chemical.

[10] According to the U.S. Patent and Trademark Office, "utility patents may be granted to anyone who invents or discovers any new and useful process, machine, article of manufacture, or composition of matter ..." As we will see in the next section, this is precisely the type of intermediate good in our model that drives endogenous growth.

is primarily concentrated on the firm side of the economy.[11]

3.1 Household

The economy is populated by a continuum of households with measure one. Each household has a unit measure of members. At the beginning of the period, all members of a household are identical and share the household's assets. During the period, the members are separated from each other, and each member receives a shock that determines the role of the member in the period. A member will be an entrepreneur with probability $\sigma_e \in [0, 1]$ and a worker with probability $\sigma_w \in [0, 1]$. They satisfy $\sigma_e + \sigma_w = 1$. These shocks are *iid* among the members and across time.

A period is divided into five stages: household's decisions, production, innovation (R&D), consumption, and investment. In the stage of household's decisions, all members of a household pool their assets: k_t units of physical capital and n_t units of equities. An equity corresponds to the ownership of a firm that is a monopolistic producer of a differentiated intermediate product. Aggregate shocks to exogenous state variables are realized. The capacity utilization rate u_t is decided, and is applied to all the capital the household possesses. Because all the members of the household are identical in this stage, the household evenly divides the assets among the members. The head of the household also gives contingency plans to each member, saying that if one becomes an entrepreneur, he or she spends s_t units of consumption goods for product developments (R&D), consumes c_t^e units of consumption goods, and makes necessary trades in the asset markets so that he or she returns to the household with k_{t+1}^e units of capital and n_{t+1}^e units of equities, and if one becomes a worker, he or she supplies l_t units of labor, consumes c_t^w units of consumption goods, sets aside i_t units of consumption goods for the investment stage, and makes necessary trades in the asset markets so that he or she returns to the household with k_{t+1}^w units of capital and n_{t+1}^w units of equities. After receiving these instructions, the members go to the market and will remain separated from each other until the investment stage.

At the beginning of the production stage, each member receives the shock whose realization determines whether the individual is an entrepreneur or a worker. Competitive firms produce final consumption goods from capital service, labor service, and specialized intermediate goods. Monopolistic firms produce specialized intermediate goods from final consumption goods; in other words, the production is roundabout. After production, a worker receives wage income, and an individual receives compensation for capital service and dividend income on equities. The government collects a uniform, lump-sum tax T_t from each member. Both a fraction $\delta(u_t)$ of capital and a fraction δ_n of products depreciate.[12] We assume that $\delta(\cdot)$ is convex in the rate of utilization: i.e., $\delta'(u_t) > 0$ and $\delta''(u_t) \geq 0$.

[11] Our implementation of Kiyotaki and Moore's financial friction is taken from Shi (2012). The production side of the economy is taken from Kung and Schmid (Forthcoming).

[12] δ_n is what Bilbiie, Ghironi, and Melitz (2012) call death shock. The assumption of exogenous exit is adopted for tractability.

The third stage in the period is R&D where entrepreneurs seek financing and undertake product development projects. We assume that an entrepreneur can transform any amount s_t units of consumption goods into $\vartheta_t s_t$ units of new products. The efficiency of product development ϑ_t is an endogenous variable (specified later), but individual households take it as given. Following Bilbiie, Ghironi, and Melitz (2012), we assume that a new product starts production in the period following invention (i.e., the adoption lag is uniform and is always constant at one).[13] With this assumption, equities of new products are traded at the same price as equities of (un-depreciated) existing products that have already paid out dividends. The goods market, the capital market, and the equity market open. Individuals trade assets to finance R&D and to achieve the portfolio of asset holdings instructed earlier by their households. The markets close at the end of this sub-period.

In the consumption stage, a worker consumes c_t^w units of consumption goods and an entrepreneur consumes c_t^e units of consumption goods. Then, individuals return to their households. In the investment stage, the head of the household collects the resources set aside by workers and uses them as inputs for investment. The capital stock at the beginning of the next period is determined by the following equation:

$$k_{t+1} = \underbrace{\left[\sigma_e k_{t+1}^e + \sigma_w k_{t+1}^w\right]}_{\text{capital before the investment stage}} + \underbrace{\left(1 - \Lambda\left(\frac{i_t}{i_{t-1}}\right)\right)\sigma_w i_t}_{\text{capital added in the investment stage}} \tag{1}$$

where $\Lambda(\cdot)$ is the investment adjustment cost function given by

$$\Lambda\left(\frac{i_t}{i_{t-1}}\right) = \frac{\bar{\Lambda}}{2}\left(g - \frac{i_t}{i_{t-1}}\right)^2$$

and g is the growth rate of the economy on the non-stochastic steady state growth path.

The instructions have to satisfy a set of constraints. First, the instructions to an entrepreneur have to satisfy the intra-period budget constraint:

$$\underbrace{c_t^e + s_t + \underbrace{p_{n,t} n_{t+1}^e + p_{k,t} k_{t+1}^e}_{\text{gross asset purchases}}}_{\text{gross expenditure}} = \underbrace{\underbrace{\Pi_t n_t}_{\text{dividend}} + \underbrace{R_t\left(u_t k_t\right)}_{\text{rental}} + \underbrace{p_{n,t}\left(1-\delta_n\right) n_t + p_{k,t}\left(1-\delta\left(u_t\right)\right) k_t}_{\text{resale value}} + \underbrace{p_{n,t}\vartheta_t s_t}_{\text{IPO}} - T_t}_{\text{gross after-tax income}} \tag{2}$$

The left-hand side is the gross total expenditure, collecting bills on consumption, R&D, and gross asset purchases, with $p_{n,t}$ denoting the price of equity and $p_{k,t}$ denoting the price of capital, respectively. The right-hand side is the gross, after-tax total income, collecting dividend income, compensation for capital service, resale values of assets, and the income from the (hypothetical) initial public offerings of new products the entrepreneur has just innovated, subtracting the lump-

[13] Comin and Gertler (2006) consider a more realistic adoption stage.

sum tax. The constraint therefore states that the total expenditure and the total after-tax income has to be balanced within a period in which an entrepreneur is separated from other members of the household. A similar constraint applies to a worker:

$$c_t^w + i_t + p_{n,t} n_{t+1}^w + p_{k,t} k_{t+1}^w = \Pi_t n_t + R_t \left(u_t k_t\right) + p_{n,t} \left(1 - \delta_n\right) n_t + p_{k,t} \left(1 - \delta \left(u_t\right)\right) k_t + W_t l_t - T_t \quad (3)$$

There are other, crucial constraints on trading of assets. That is, an entrepreneur can sell at most a fraction θ of new equities for products she has just innovated but has to retain the rest of equities by herself. In addition, she can sell at most a fraction ϕ_t of both existing equities and existing capital to others in the asset markets but has to retain the rest by herself. Effectively, these constraints introduce lower bounds to equity holding and capital holding of an entrepreneur at the closing of the markets:

$$n_{t+1}^e \geq \underbrace{\left(1 - \theta\right) \vartheta_t s_t}_{\text{new equities required to retain}} + \underbrace{\left(1 - \phi_t\right) \left(1 - \delta_n\right) n_t}_{\text{existing equities required to retain}} \quad (4)$$

$$k_{t+1}^e \geq \underbrace{\left(1 - \phi_t\right) \left(1 - \delta \left(u_t\right)\right) k_t}_{\text{existing capital required to retain}} \quad (5)$$

ϕ_t is an exogenous, random variable representing shocks to asset liquidity.[14] Similar constraints apply to workers, i.e., $n_{t+1}^w \geq \left(1 - \phi_t\right) \left(1 - \delta_n\right) n_t$ and $k_{t+1}^w \geq \left(1 - \phi_t\right) \left(1 - \delta \left(u_t\right)\right) k_t$, but we omit them because they do not bind in the equilibrium. There are non-negativity constraints for u_t, s_t, c_t^e, l_t, i_t, c_t^w, n_{t+1}^w, and k_{t+1}^w, but we omit them too for the same reason.

We view the equity market and the capital market as collectively representing the financial system, because these markets, albeit in a highly stylized manner, intermediate between investors (entrepreneurs) and capital providers (workers). In addition, as in the actual economy, our model's growth potential hinges on the efficiency of those markets to transfer funds from those who are willing to supply them to those who need them for innovations. The liquidity shock is a potential clog in the fund supply conduits, and we use its fluctuation to capture variation in financial conditions we documented in the previous section.

Let q_t denote the vector of endogenous, individual state variables, i.e., $q_t = \left(n_t, k_t, i_{t-1}\right)$. The head of the household chooses instructions to its members to maximize the value function defined as

$$v\left(q_t; \Gamma_t, \Theta_t\right) = \max \left\{ \sigma_e \log\left(c_t^e\right) + \sigma_w \left[\log\left(c_t^w\right) - \psi_t \frac{l_t^{1+\zeta}}{1+\zeta} \right] + \beta_t E_t \left[v\left(q_{t+1}; \Gamma_{t+1}, \Theta_{t+1}\right) \right] \right\} \quad (6)$$

[14] Brunnermeier et al. (2012) refer to this type of liquidity as *market liquidity*. Since our model does not feature irreversibilities, physical and intangible capitals are also *technologically liquid*.

subject to (1), (2), (3), (4), (5), and

$$n_{t+1} = \sigma_e n_{t+1}^e + \sigma_w n_{t+1}^w,$$

β_t is a subjective time discount factor, and ψ_t is a coefficient affecting the labor disutility schedule, both of which are common across households and are exogenous random variables. Γ_t is the vector of endogenous, aggregate state variables, i.e., $\Gamma_t = (N_t, K_t, I_{t-1})$, where N_t is the mass of products available in the economy, K_t is the capital stock in the economy, and I_{t-1} is the investment level in the previous period. Θ_t is the vector of exogenous state variables.

As in Shi (2012), we will restrict our attention to the case in which $1 < p_{n,t}\vartheta_t < 1/\theta$ always hold. The first inequality, $1 < p_{n,t}\vartheta_t$, implies that R&D is a good business, because marginal costs of product developments are smaller than marginal revenues of product developments. The second inequality, $p_{n,t}\vartheta_t < 1/\theta$, implies that the entrepreneur must prepare a down payment, because the amount of product development costs that the entrepreneur can finance by issuing equities is smaller than the total costs. These two conditions jointly imply that an entrepreneur's liquidity constraints (4) and (5) must be binding at the optimum. Please see the appendix for the formal discussion.

First-order optimality conditions are derived in the appendix. The conditions concerning worker's choice variables are completely standard:

$$\psi_t l_t^\zeta = W_t \left(\frac{1}{c_t^w}\right) \tag{7}$$

$$1 = p_{k,t}\left(1 - \Lambda\left(\frac{i_t}{i_{t-1}}\right) - \Lambda'\left(\frac{i_t}{i_{t-1}}\right)\frac{i_t}{i_{t-1}}\right) + E_t\left[\left(\beta_t \frac{c_t^w}{c_{t+1}^w}\right) p_{k,t+1}\Lambda'\left(\frac{i_{t+1}}{i_t}\right)\left(\frac{i_{t+1}}{i_t}\right)^2\right] \tag{8}$$

(7) equates marginal disutility of labor to marginal utility of receiving wage income, while (8) equates the cost of investment measured in consumption goods to the benefit of investment, which incorporates not only the value of capital created in the current period, but also investment's dynamic effects on future investment adjustment costs. These conditions are standard because a worker's liquidity constraints are not binding in the equilibrium. We also show that the marginal benefit of adding an asset to the household's portfolio is equated to the opportunity cost of buying it using a worker's budget;

$$\beta_t E_t\left[\frac{\partial v_{t+1}}{\partial n_{t+1}}\right] = p_{n,t}\left(\frac{1}{c_t^w}\right) \tag{9}$$

$$\beta_t E_t\left[\frac{\partial v_{t+1}}{\partial k_{t+1}}\right] = p_{k,t}\left(\frac{1}{c_t^w}\right) \tag{10}$$

In other words, a worker's marginal utility of consumption is an appropriate measure to evaluate asset values. Again, this is because a worker's liquidity constraints are not binding in the equilibrium.

An entrepreneur's marginal utility is related to worker's as follows:

$$\left(\frac{1}{c_t^e}\right) = \left(\frac{\vartheta_t\left(1-\theta\right)}{1-\theta p_{n,t}\vartheta_t}\right) p_{n,t} \left(\frac{1}{c_t^w}\right) \tag{11}$$

This equation is essentially an optimality condition for product developments. The intuition is the following. An entrepreneur can increase the utility by consuming the last unit of her disposable income (the left-hand side). If, however, she devotes the same resource to product development, she can create $\vartheta_t / \left(1-\theta p_{n,t}\vartheta_t\right)$ units of new products, which is the efficiency of converting consumption goods to new products multiplied by the reciprocal of the down payment. Among the developed products, a fraction $(1-\theta)$ is unsold in the market and therefore added to the household's asset portfolio. Lastly, since the household's subjective valuation of an equity is equal to the opportunity cost of buying it using a worker's budget, the right-hand side represents the benefit expected when the entrepreneur uses the last unit of her disposable income for product development. Equation (11) therefore says that the household should find the entrepreneur's consumption and product developments indifferent at the margin. The same condition can be conveniently expressed as follows:

$$\left(\frac{1}{c_t^e}\right) = \left(1+\lambda_t^e\right)\left(\frac{1}{c_t^w}\right), \tag{12}$$

where λ_t^e is defined as

$$\lambda_t^e = \frac{p_{n,t}\vartheta_t - 1}{1-\theta p_{n,t}\vartheta_t}. \tag{13}$$

λ_t^e is the variable Shi (2012) calls the liquidity services. Our assumption $1 < p_{n,t}\vartheta_t < 1/\theta$ implies that the liquidity services are always positive, implying that entrepreneurs consume less than workers in the equilibrium. This is because freeing up a unit of resource in the entrepreneur's budget constraint is more valuable to the household than freeing up a unit of resource in the worker's budget constraint, as product developments generate profits. The liquidity services measure the relative importance to the household between the incremental resource given to an entrepreneur and the incremental resource given to a worker.

Prices of equity and capital are determined by the following equations:

$$p_{n,t} = E_t\left[\left(\beta_t \frac{c_t^w}{c_{t+1}^w}\right)\left(\Pi_{t+1} + p_{n,t+1}\left(1-\delta_n\right) + \sigma_e\lambda_{t+1}^e\left[\Pi_{t+1} + p_{n,t+1}\left(1-\delta_n\right)\phi_{t+1}\right]\right)\right] \tag{14}$$

$$p_{k,t} = E_t\left[\left(\beta_t \frac{c_t^w}{c_{t+1}^w}\right)\left(R_{t+1}u_{t+1} + p_{k,t+1}\left(1-\delta\left(u_{t+1}\right)\right) + \sigma_e\lambda_{t+1}^e\left[R_{t+1}u_{t+1} + p_{k,t+1}\left(1-\delta\left(u_{t+1}\right)\right)\phi_{t+1}\right]\right)\right]. \tag{15}$$

Equation (14) says that the price of equity reflects not only the present discounted value of future cash flow but also the present discounted value of future liquidity services. The liquidity services are incorporated into the equilibrium price because a product delivers dividend income to its shareholders, and, the equity is saleable up to a certain fraction in the equity market, both of which are attractive to the household because they provide liquidity to entrepreneurs. An analogous intuition applies to (15).

Finally, the first order optimality condition for capacity utilization rate is given by

$$R_t + p_{k,t}\left(-\delta'\left(u_t\right)\right) + \sigma_e \lambda_t^e \left(R_t + p_{k,t}\phi_t\left(-\delta'\left(u_t\right)\right)\right) = 0 \tag{16}$$

The head of the household cares about not only the usual tradeoff between revenue (the first term) and depreciation (the second term) but also how much liquidity she can provide to entrepreneurs with capital (the third term).

3.2 Final goods sector

There is a representative firm that uses capital service KS_t, labor L_t, and a composite of intermediate goods G_t to produce the final good according to the production technology

$$Y_t = \left(\left(KS_t\right)^{\alpha}\left(A_t L_t\right)^{1-\alpha}\right)^{1-\xi} G_t^{\xi}, \tag{17}$$

where the composite G_t is defined as

$$G_t = \left[\int_0^{N_t} X_{i,t}^{\frac{1}{\nu}} di\right]^{\nu}.$$

Here, $X_{i,t}$ is intermediate good $i \in [0, N_t]$, α is the capital share, ξ is the intermediate goods share, and ν is the parameter affecting the elasticity of substitution between the intermediate goods. A_t is the exogenous, neutral productivity shock. The firm maximizes profits defined as

$$Y_t - R_t\left(KS_t\right) - W_t L_t - \int_0^{N_t} P_{i,t} X_{i,t} di,$$

where $P_{i,t}$ is the price per unit of intermediate good i, which the final goods firm takes as given. Solving the cost minimization problem of purchasing intermediate goods leads to the downward-sloping demand function:

$$X_{i,t} = \left(\frac{P_{i,t}}{P_{G,t}}\right)^{\frac{\nu}{1-\nu}} G_t, \tag{18}$$

where $P_{G,t}$ is the price index defined as

$$P_{G,t} = \left[\int_0^{N_t} P_{i,t}^{\frac{1}{1-\nu}} di\right]^{1-\nu}.$$

The total expenditure on intermediate goods is given by

$$\int_0^{N_t} P_{i,t} X_{i,t} di = P_{G,t} G_t.$$

The firm's first-order optimality conditions are standard:

$$R_t = (1 - \xi)\, \alpha \frac{Y_t}{KS_t}, \tag{19}$$

$$W_t = (1 - \xi)\,(1 - \alpha)\, \frac{Y_t}{L_t}, \tag{20}$$

$$P_{G,t} = \xi \frac{Y_t}{G_t}. \tag{21}$$

3.3 Intermediate goods sector

Production is roundabout, and hence the marginal cost of producing an intermediate good is unity. The producer chooses its price $P_{i,t}$ to maximize the profits defined as

$$\Pi_{i,t} \equiv \max_{P_{i,t}} (P_{i,t} - 1) \left(\frac{P_{i,t}}{P_{G,t}} \right)^{\frac{\nu}{1-\nu}} G_t.$$

Solving this problem leads to the optimal markup pricing,

$$P_{i,t} = \nu. \tag{22}$$

Since prices are symmetric, so are production levels and profits. Let X_t denote the symmetric production level, i.e., $X_t = X_{i,t}$ for all $i \in [0, N_t]$, and let Π_t denote the symmetric profits, i.e., $\Pi_t = \Pi_{i,t}$ for all $i \in [0, N_t]$. Profits are paid out to shareholders as dividends.

3.4 Product development technology

We assume that the technology coefficient of product development involves both knowledge spillover á la Romer (1990) and a congestion externality effect capturing decreasing returns to scale in the innovation sector

$$\vartheta_t = \frac{\chi_t N_t}{(\sigma_e s_t)^{1-\eta} (N_t)^{\eta}}, \tag{23}$$

where $\eta \in [0, 1]$ is the elasticity of new intermediate goods with respect to R&D. χ_t represents an exogenous, sector-specific productivity shock in the innovation sector. N_t's transition rule is given by

$$N_{t+1} = (1 - \delta_n)\, N_t + \vartheta_t\, (\sigma_e s_t)\,.$$

Notice that although the product entry rate (the second term on the right-hand side) is decreasing returns to scale in aggregate R&D due to the congestion externality, the same term, and hence the right-hand side as a whole as well, is homogeneous of degree one in N_t and s_t because of the knowledge spillover. The growth rate of N_t therefore depends on (except for the sector-specific productivity shock) only the ratio of aggregate R&D spending to the mass of products available in the economy. Hence, as long as these two variables grow proportionally, the innovation does not have to slow down, even in the long run. This is an important insight in the endogenous growth literature. An equally important implication, especially for our purposes, is that the model can link the trend and the cycle. Specifically, since the model's growth mechanism relies on a virtuous circle between R&D and knowledge spillover, a recession might leave permanent scars on the economy if it causes a severe disruption in R&D.

3.5 Government

The government spends a fraction τ_t of the value-added output $\mathcal{Y}_t$, which is defined as the gross output minus intermediate inputs

$$\mathcal{Y}_t \equiv Y_t - N_t X_t.$$

We assume that the government keeps the balanced-budget:

$$\tau_t \mathcal{Y}_t = T_t.$$

τ_t is an exogenous, random variable.

3.6 Equilibrium

The competitive equilibrium is defined in a standard way. Market clearing conditions for production factors are

$$KS_t = u_t K_t,$$

$$L_t = \sigma_w l_t.$$

Goods market clearing condition is

$$Y_t = \sigma_e c_t^e + \sigma_w c_t^w + \sigma_w i_t + \sigma_e s_t + N_t X_t + T_t.$$

Asset markets clearing conditions are

$$N_t = n_t,$$

$$K_t = k_t,$$

at the beginning of a period and

$$N_{t+1} = \sigma_e n_{t+1}^e + \sigma_w n_{t+1}^w,$$

$$(1 - \delta(u_t)) K_t = \sigma_e k_{t+1}^e + \sigma_w k_{t+1}^w,$$

at the end of the production stage.

Let us discuss important equilibrium properties. Using (22) and the symmetry between products, we find that the price index of the intermediate goods composite is given by

$$P_{G,t} = N_t^{1-\nu}\nu. \tag{24}$$

Using (17), (21), and (24), we find that the final goods production is given by

$$Y_t = \left(\frac{\xi}{\nu}\right)^{\frac{\xi}{1-\xi}} (KS_t)^{\alpha} (A_t L_t)^{1-\alpha} (N_t)^{\frac{\nu\xi-\xi}{1-\xi}}. \tag{25}$$

Following Kung and Schmid (Forthcoming), we make the parameter restriction $\alpha + \frac{\nu\xi-\xi}{1-\xi} = 1$. An advantage of this assumption is that it leads to a production function that resembles the standard neoclassical one with labor augmenting technology

$$Y_t = (KS_t)^{\alpha} (Z_t L_t)^{1-\alpha},$$

where the equilibrium productivity measure is given by

$$Z_t = \left(\bar{A}\right)(A_t N_t),$$

and $\bar{A} \equiv \left(\frac{\xi}{\nu}\right)^{\frac{\xi}{(1-\xi)(1-\alpha)}} > 0$ is a constant. The equilibrium productivity process thus contains a component driven by two factors: the exogenous productivity shock, A_t, and an endogenous trend component, N_t. The second component reflects a well-known variety effect: i.e., the expansion of product varieties allows more efficient use of labor and capital in final-goods production because, as is clear from (24), the price of the intermediate-goods composite relative to the final goods decreases with product varieties.

We discuss the national income accounting. Rearranging the goods market clearing condition, we find an identity

$$\mathcal{Y}_t = \sigma_e c_t^e + \sigma_w c_t^w + \sigma_w i_t + \sigma_e s_t + T_t, \tag{26}$$

The value added output is the sum of consumption, investment in capital, investment in product developments, and government spending. Another approach to the aggregate value added output is from income. Using (19), (20), and (21), we find

$$Y_t = R_t\left(u_t K_t\right) + W_t\left(\sigma_w l_t\right) + P_{G,t} G_t.$$

Since the revenues for the intermediate goods firms are decomposed into costs and profits, we find

$$\mathcal{Y}_t = R_t\left(u_t K_t\right) + W_t\left(\sigma_w l_t\right) + N_t \Pi_t. \tag{27}$$

The value added output is the sum of the compensation for capital service, the compensation for labor service, and profits.

Finally, the Cobb-Douglas production function (17) and the constant markup (22) imply that the relation between the gross output and the value added output is linear:

$$\mathcal{Y}_t = \left(1 - \frac{\xi}{\nu}\right) Y_t.$$

In the appendix, we provide the derivation and the summary of the equilibrium conditions.

3.7 Structural Shocks

There are six structural shocks, β_t, ϕ_t, χ_t, A_t, τ_t, and ψ_t, in our model, each of which is modeled as an AR(1) process with *iid* innovation. Hence, the generic specification of our shocks is

$$\log \frac{\varsigma_t}{\varsigma} = \rho_\varsigma \log \frac{\varsigma_{t-1}}{\varsigma} + \sigma_\varsigma \varepsilon_\varsigma,$$

where ρ_ς and σ_ς are the persistence and standard deviation of the stochastic process. The innovation or shock ε_ς is assumed to follow a normal standard distribution.

4 Results

Before discussing in detail the main results from our model, we explain how we choose the parameters.

4.1 Estimation

We take a fairly conservative approach regarding the parameterization/estimation of our model. We tie our hands by setting most structural parameters to either values used elsewhere or to match some incontrovertible ratio in the data. This means that our estimation strategy puts the emphasis on the structural shocks.

The first panel of Table 1 reports the parameters that are fixed during estimation. Following Shi

Table 1: Parameter Values

Fixed			Estimated		
Parameter	Calibration	Reference	Parameter	Mode	Priors
β	0.92	Match empirical moments	δ''/δ'	2.73	G (1.0,0.5)
ζ	1	Comin and Gertler (2006)	$\overline{\Lambda}$	0.02	G (3.0,1.0)
η	0.8	Comin and Gertler (2006)	ρ_β	0.22	B (0.5,0.2)
σ_e	0.06	Shi (2012)	ρ_ϕ	0.99	B (0.5,0.2)
δ_n	0.03	Bilbiie et al. (2012) and others	ρ_χ	0.22	B (0.5,0.2)
ν	1.6	Comin and Gertler (2006)	ρ_a	0.53	B (0.5,0.2)
δ_k	0.1	Match empirical moments	ρ_τ	0.54	B (0.5,0.2)
θ	0.2	Del Negro et al. (2011); Shi (2012)	ρ_ψ	0.10	B (0.5,0.2)
ϕ	0.2	Del Negro et al. (2011); Shi (2012)	σ_β	0.05	IG (0.2,0.1)
τ	0.2	Match gov spending to GDP	σ_ϕ	0.38	IG (0.2,0.1)
δ'	0.19	Pinned down by equil. condition	σ_χ	0.04	IG (0.2,0.1)
α	0.36	Labor Share	σ_a	0.03	IG (0.2,0.1)
χ	0.47	Match empirical moments	σ_τ	0.04	IG (0.2,0.1)
u	1	Normalization	σ_ψ	0.03	IG (0.2,0.1)

G: Gamma distribution, B: Beta distribution, IG: Inverse Gamma distribution. Numbers in parenthesis are mean and standard deviation.

(2012), we set the share of liquidity constrained agents (entrepreneurs) σ_e at 6 percent per quarter. We set the product depreciation rate δ_n at 3 percent per quarter, following the literature (Bilbiie, Ghironi, and Melitz (2012); Comin and Gertler (2006); and Kung and Schmid (Forthcoming)). The steady state neutral productivity shock and the steady state labor disutility shock are set so that both the steady state gross output after detrending and the steady state labor hours are equal to one. These are only normalizations. The steady state time discount rate, the steady state capital depreciation rate δ_k, and the steady state sector-specific productivity shock are jointly calibrated. The empirical targets are mean GDP growth rate, mean investment to GDP ratio, and mean R&D to GDP ratio.

The second panel in turn reports the mode of the posterior distribution of the estimated parameters. We use gamma distribution priors for the elasticity of capital utilization (δ''/δ') and the adjustment cost of investment $\left(\overline{\Lambda}\right)$. The mean and standard deviations are $\{1, 0.5\}$ and $\{3, 1\}$, respectively. For the persistence of the stochastic processes, we choose a beta distribution with mean 0.5 and standard deviation 0.2. The prior for the volatility of the structural shocks is an inverse gamma distribution with parameters 6 and 1.

We estimate our model using quarterly data on output, consumption, investment, wages, labor, and data on intangible capital.[15] For the first three observables, nominal values (from NIPA) were deflated using the implicit GDP deflator. Wages correspond to nominal compensation per hour in

[15] The use of intangible capital data reflects our view of products in the model. We define them broadly. In addition, we believe that products are able to distinguish themselves from other products not only by the formal patent system but also by informal protections surrounding trade secrets, brand images, business models, and so on. Such a consideration leads us to use an inclusive measure.

the nonfarm business sector deflated by the implicit GDP deflator. Labor is the ratio of hours of all persons in the nonfarm business sector to the civilian noninstitutional population. For the last observable, we rely on the series reported in Nakamura (2003). Without going into the details, Nakamura argues that a more accurate portrayal of intangible capital in the economy is given by twice the measure of software plus twice the value of R&D (both taken from NIPA) plus a measure of advertisement spending (compiled by the advertising agency McCann and Erickson). We adjust output and investment to reflect this alternative (and broader) measure of R&D (the annual series was interpolated using the algorithm of Fernandez (1981) with NIPA's R&D quarterly series as the reference entry). The sample covers 1970.Q1 - 2011.Q4. Except for labor, all variables are expressed in growth rates.

Before analyzing the Great Recession, we briefly discuss the impact of liquidity changes in our model (Figure 5). Following an adverse liquidity innovation (a decrease in ϕ_t), both equity and capital become less liquid. Entrepreneurs scale down their product development projects because they struggle to fund their businesses, as cashing out assets is now not as easy as before. Weak innovative activities have detrimental impacts on future innovations through knowledge spillover. R&D spending in subsequent periods further declines, resulting in prolonged weak growth in the economy. As we explained in Section 2, the U.S. economy did experience a slowdown (growth below average) in the number of patents granted during the recession. To the extent that these patents eventually materialize as intermediate goods for production, the decline in R&D and hence in the trend under the tight liquidity condition are consistent with the data.

4.2 A look at the Great Recession

Figure 6 shows the smoothed paths for the stochastic processes (ς_t) around the Great Recession (the red dot indicates 2008.Q3). Two crucial observations emerge from these figures. First, the dynamics of the discount factor point to a large change in the second half of 2008, which most likely reflects the households' efforts to de-leverage (we will get back to this issue). The government spending shocks signal low demand during the recession. More important for our purposes, the liquidity condition in the asset market significantly deteriorated. Recall that a decline in ϕ_t means that people can sell a smaller fraction of their physical and intangible assets. Indeed, the worst liquidity shock coincided with the failure of Lehman Brothers. Our estimates suggest that the financial conditions started to improve in 2010. Low aggregate demand and adverse financial conditions translate into weakness in labor market. The adverse labor supply shock (ψ_t) further amplifies the bad situation in this market.

Figure 6 provides an interesting account of the worsening conditions in the financial markets in 2008. In particular, the dynamics of ϕ_t indicate that tightening in the credit markets started in mid-2007 (presumably due to the first wave of foreclosures). Interestingly, the timing of the peak in our measure of liquidity coincides with the peak of the credit boom (Ivashina and Scharfstein

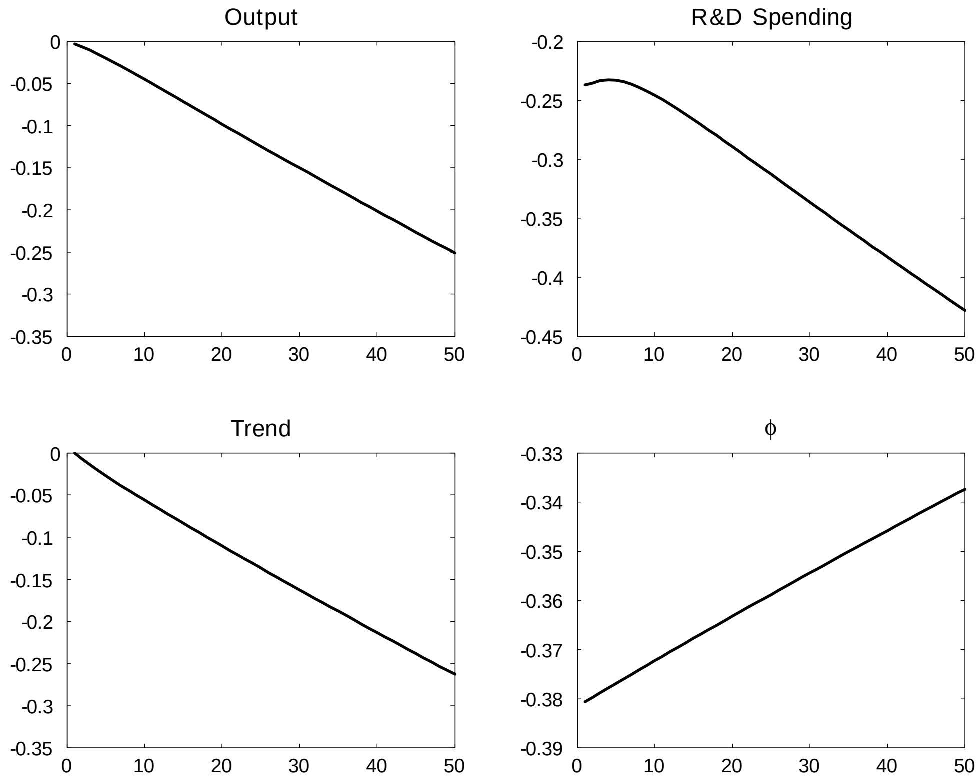

Figure 5: Impulse Responses to a Decrease in Liquidity

(2010)) and the highest private equity investment (Figure 4). Our measure reveals that the financial crisis gained substantial momentum following the demise of Bear Stearns in early 2008, particularly after Lehman Brothers' collapse later in the year. Credit conditions remained tight (although improving) through 2009. It is only in mid-2010 when the financial markets showed signals of more favorable financial markets. As Figure 7 shows, our measure of liquidity not only agrees with the anecdotal descriptions of the crisis but also tracks actual measures very closely. Indeed, liquidity as predicted by our exercise moves in surprising coordination with the (negative) margin on S&P 500 futures (red line) reported in Section 2. The correlation coefficient is 0.93. The figure also shows (green line) the path of the (first principal component) financial shock estimated by Stock and Watson (2012). As before, this shock points to adverse financial conditions around the same time predicted by our model. Although the subsequent recovery is faster and stronger than in our measure, we find a strong correlation (around 0.82) between these two measures interesting. These findings are quite remarkable and a favorable validation of our approach if we consider that we used no financial data to estimate the model.

The decline in the discount rate shock in the middle of the Great Recession (Figure 6) is at first glance puzzling, because in a standard RBC model, a fall in the discount rate would imply a counterfactual improvement in consumption. However, a negative discount rate shock in our model economy is also associated with a negative wealth effect, which counterbalances the aforementioned intertemporal substitution effect. This is because the temporary improvement

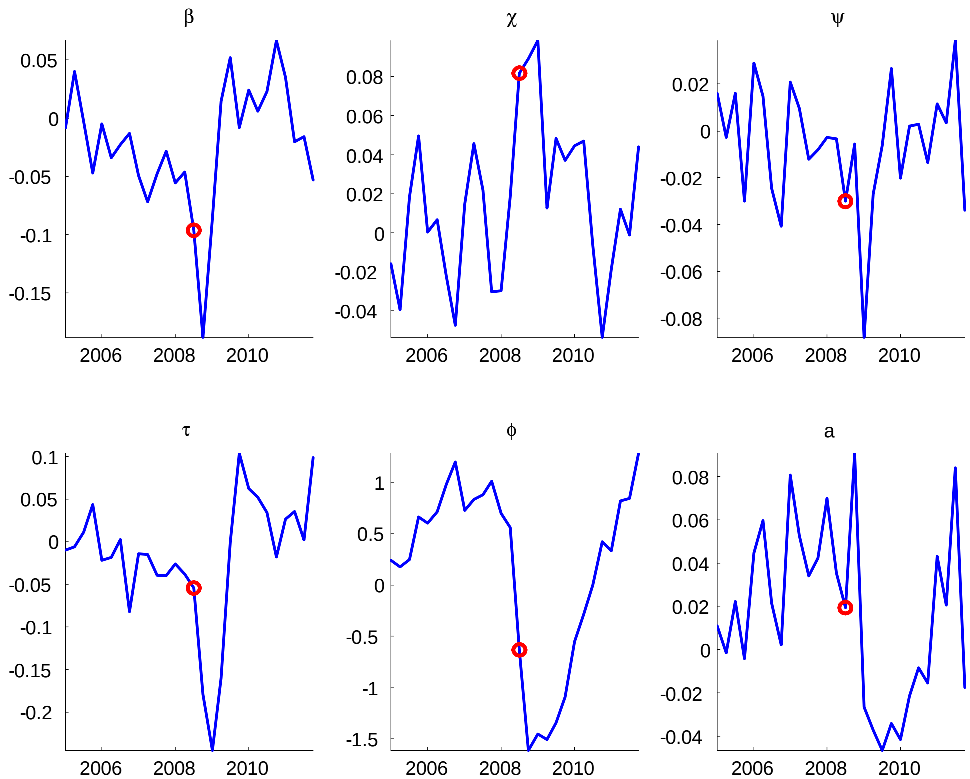

Figure 6: Smoothed Stochastic Processes

in consumption comes crucially at the expense of R&D spending in our model (unlike in the RBC model, where an increase in consumption comes solely at the expense of a contraction in investment), and the lower pace of innovation in turn implies a downward kick to the economy's trend. Notice that the knowledge spillover backfires in this situation, amplifying the mechanism.

To assess the severity of the financial shock and its implications for the shift in the GDP trend, we try alternative scenarios about the evolution of this shock following the collapse of Lehman Brothers. Our first counterfactual simulation (dashed red lines in Figure 8; solid-blue lines correspond to actual paths) assumes that the financial markets remained frozen at their worst state in 2008, which happened in the fourth quarter (this is implemented by assuming that ϕ_t was fixed at its value in 2008.Q4). The important message from this counterfactual is that improving conditions in the financial sector played a critical role in the post-crisis recovery. Although the fictitious economy followed a path close to the actual economy in early 2009, it is clear that lingering adverse financial conditions would have led to a deeper and longer recession lasting well into the end of our sample.

We also find quite interesting (and suggestive) that labor remains contracted in our simulation when the financial friction remains at its worst state. The reason behind the labor path is as follows. Workers in the counterfactual simulation do not spend lots of money on purchasing assets, because assets are illiquid and, as a consequence, there are not many assets sold in the market. This means that a larger amount of liquidity remains in workers' hands. They consume, rather

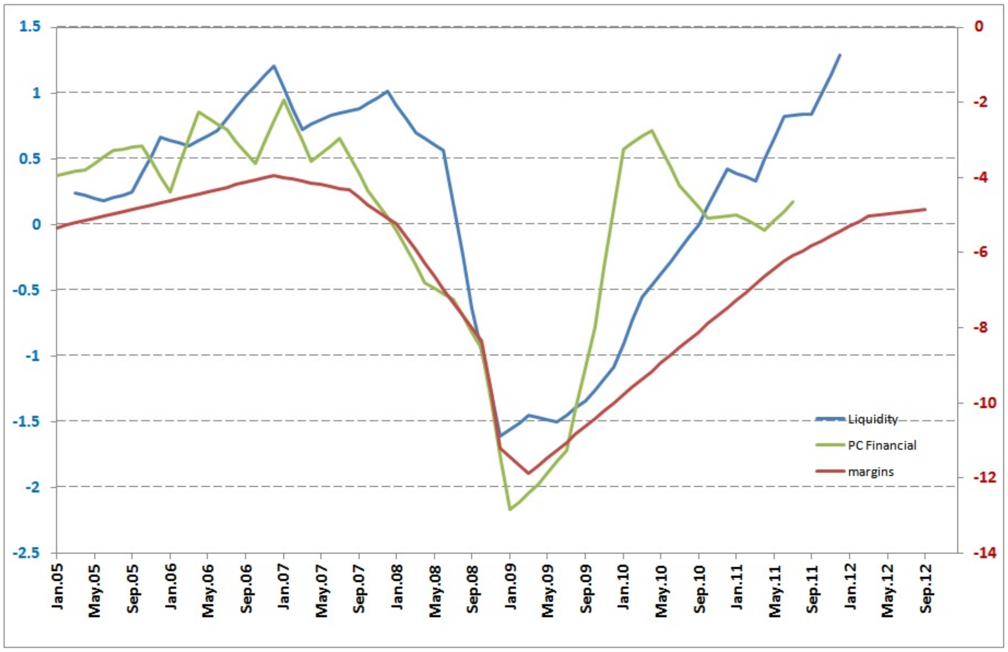

Figure 7: Estimated Liquidity, Margins on S&P 500 Futures, and Stock and Watson's Financial Shock

than invest, much of this windfall of liquidity because growth prospects are dismal. This then generates a sort of "income effect," weakening labor supply condition by increasing the marginal rate of substitution. This cross-sectional resource misallocation (i.e., the liquidity-constrained entrepreneurs cannot access to the funds they long for, but much of the resource is stuck in the workers' hands and is consumed by them) explains why the labor market remains constricted in the counterfactual simulation. In short, this is a symptom of the sclerotic financial market. But our counterfactual simulation shows that dynamic consequence of the adverse liquidity condition is even more severe, i.e., smaller investment in the current period reduces the stock of physical capital in the next period, which further reduces investment in the future. This vicious cycle shows no tendency to slow down unless the liquidity condition recovers, leading to a wide gap between the actual path and the counterfactual simulation.

The dynamics of research and development are not very different between the counterfactual simulation and the actual path, at least in the time span we are concerned about. This may be puzzling at first, since other things being equal, the liquidity shock has a direct impact on entrepreneur's behavior. But the reason is actually simple; adverse real shocks hit the research and development sector right after the Lehman shock. We will come back to this point later.

Figure 9 in turn displays the counterfactual scenario for the variables of interest in levels (we use 2005.Q1 as the reference point to compute the series). Had the financial markets remained frozen, our simulations indicate that GDP would have been 20 percentage points below its actual level by the end of our sample (2011.Q4). Note that this astonishing break in the GDP trend is a consequence of the endogenous growth feature of our model. Indeed, this scenario suggests that

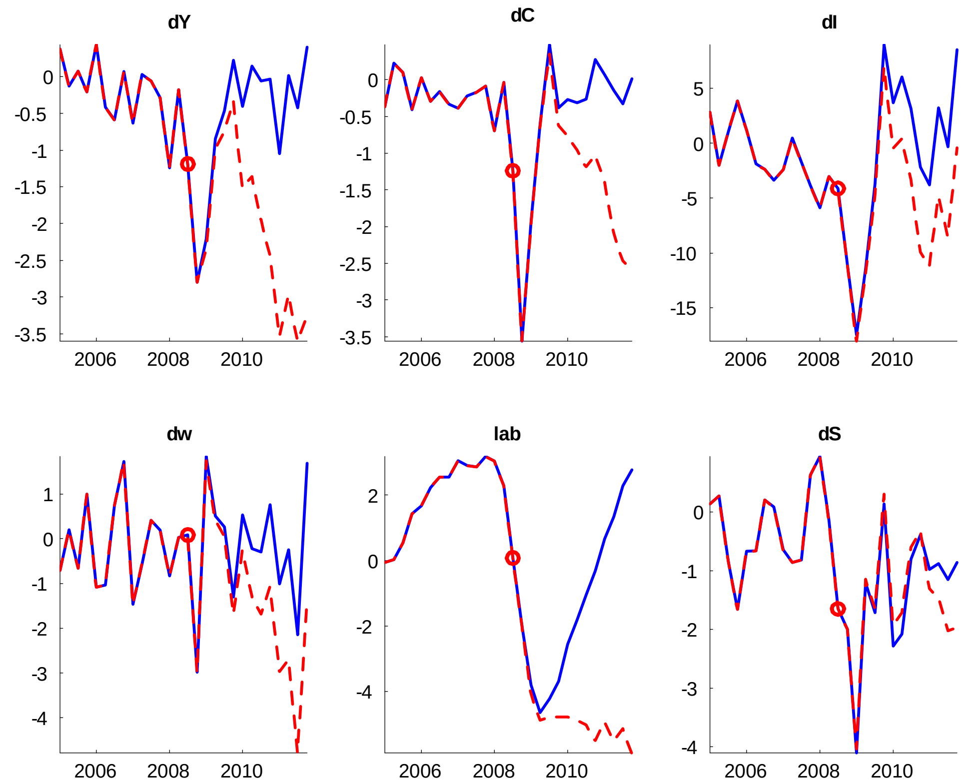

Figure 8: Counterfactual Scenario: Liquidity Constraint Stuck at its Lowest Level in 2008

a sclerotic financial sector could have easily led to a collapse of the economy with a speed and severity comparable to those experienced during the Great Depression.

Another way to study the degree of financial tightening and its impact on the Great Recession is as follows. Imagine that starting in 2009.Q1, the financial shocks ($\varepsilon_{\phi,t}$) follow their estimated paths but the other shocks are replaced by random draws with replacement from their empirical distribution.[16] This exercise describes the dynamics of an average economy except that it is buffeted by the actual liquidity innovations. The resulting paths for the levels of several variables in the model are plotted in Figure 10. We think that the figure speaks for itself. Namely, the more favorable liquidity conditions would have eventually led growth to positive numbers and hence brought the economy close to the trend prevalent in 2011, albeit with some delays. Indeed, this counterfactual suggests that the influence of the non-liquidity shocks were crucial for the speed of the recovery but not for the recovery itself. By the end of the sample, the better outlook in financial conditions was enough to get the counterfactual output trend (red dashed line) close to the actual data. Interestingly, a large part of the dynamics of the labor market is accounted for by the financial shock.

[16] Suppose $\{\varepsilon_{\bullet,t}\}_{t=1}^{T}$ denotes the collection of estimated shocks. The simulation proceeds by randomly drawing with replacement from this collection of shocks for all disturbances except $\varepsilon_{\phi,t}$. These draws then replace the estimated ones from 2009.Q1 and beyond. The simulation is repeated many times. The figures plot the average across all of these repetitions.

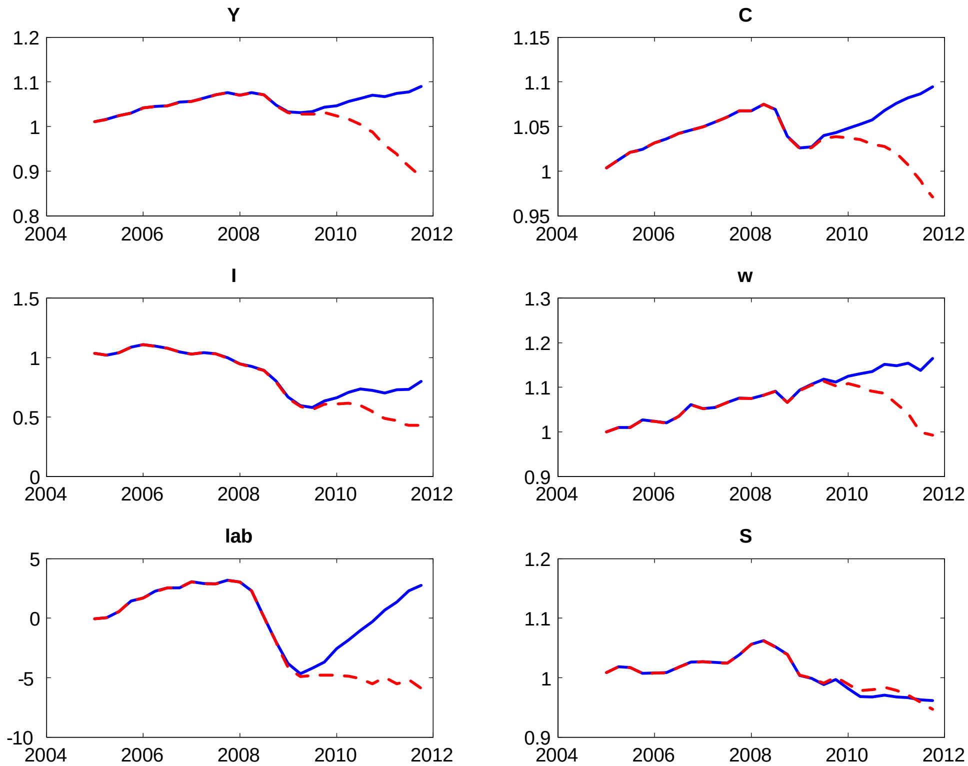

Figure 9: Counterfactual Scenario with Variables in Levels and Liquidity frozen at its 2008.Q4 level

As we anticipated, research and development is not crushed after the Lehman's collapse in the current counterfactual simulation. This result indicates that the research and development sector was hammered twice during the Great Recession. First, it was pounded by the credit crunch following Lehman's failure and, second, it was pounded again by real shocks. The impact of the latter was milder and short-lived as compared with the liquidity shock.

An alternative way to evaluate the role of liquidity is to back up a counterfactual path that would leave the trend of the economy growing at its steady state rate. Figure 11 displays the results from this exercise. The liquidity paths are plotted in the upper panel while the bottom panel contains the actual and counterfactual trends. There are two striking findings. First, this simulation suggests that the Great Recession could have been averted if the financial crisis had been averted. This result further underlines the role of the liquidity shock in the Great Recession. Second, this simulation suggests that it would have sufficed that liquidity returned to its normal levels for the economy to remain near its historical trend. Notice that the counterfactual liquidity path still experiences a drop (but not a crush) that just ends the preceding liquidity boom. This means that the other structural disturbances had a small but positive impact on the trend. This is consistent with the results in Figure 10, in which we see that the other structural shocks expedited the recovery.

We ask now at what point in time conditions became adverse to the point that the downturn

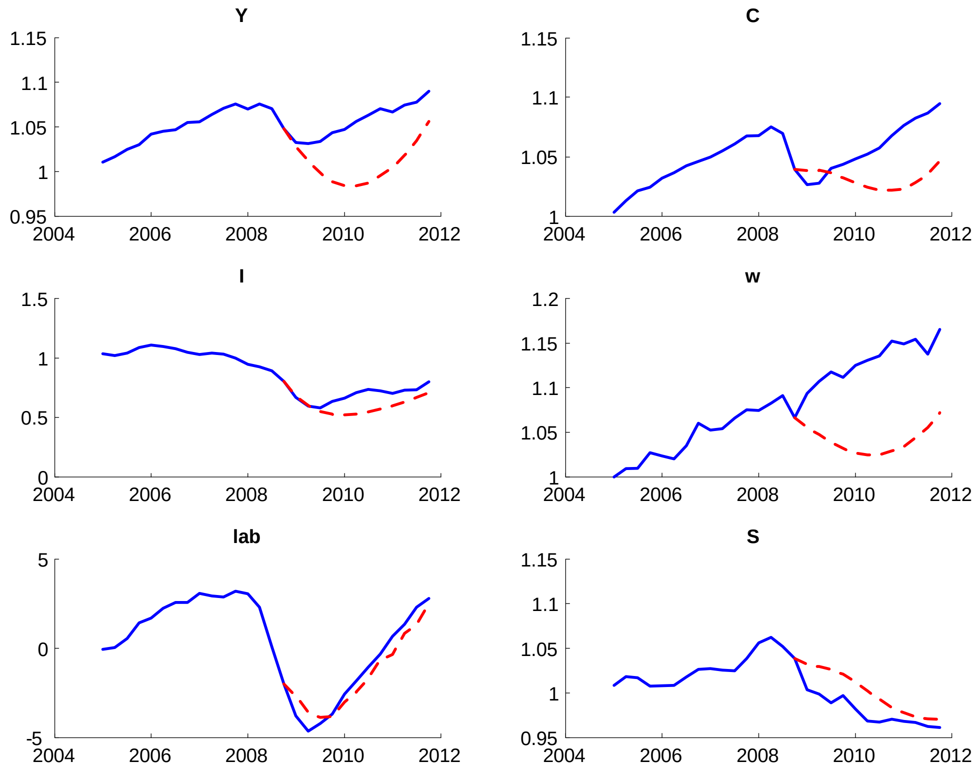

Figure 10: Counterfactual with Liquidity Following Actual Path and Other Shocks are Random

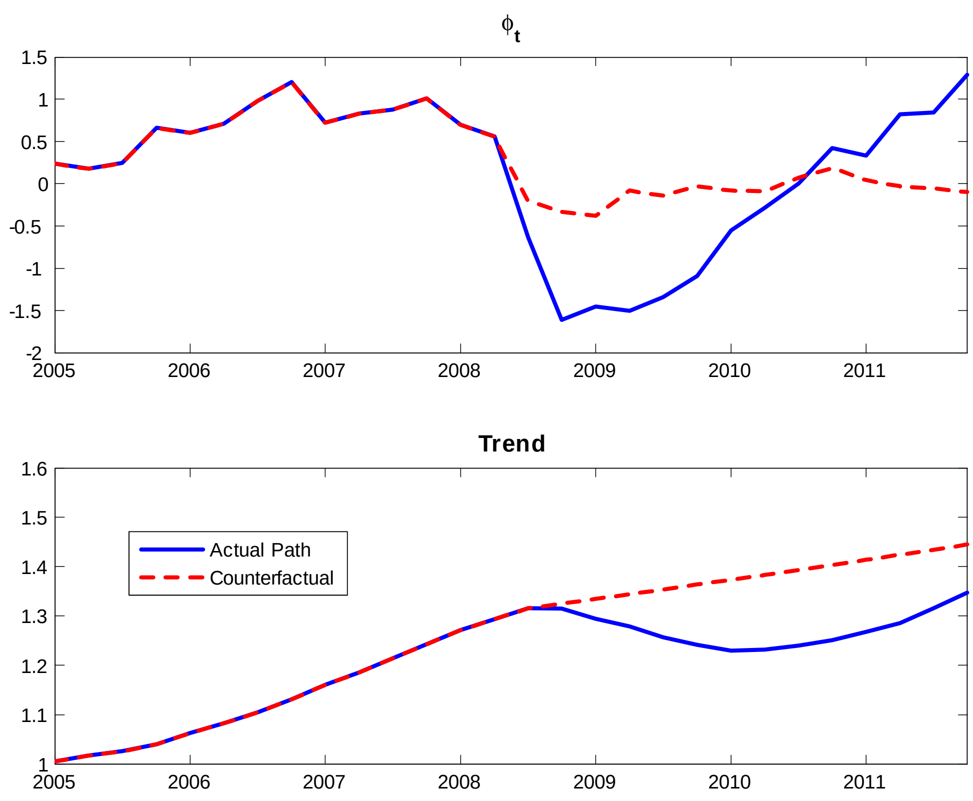

Figure 11: Liquidity and Trends

was inevitable. To this end, we simulate our model assuming that the innovations to all structural shocks, $\varepsilon_{\cdot,t}$, are randomly drawn from their distributions starting at different points in time. The idea is to assess when the innovations were bad enough to pull the economy into recession even though from that point and beyond the economy is buffeted by average shocks. In other words, by sequentially rolling back the timing of randomization, we can assess the "marginal effect" of structural shocks that hit the economy in a particular point in time, and by doing so, we can ask which vintage of shocks push the economy over the edge of a cliff. The blue dotted lines with the steepest slope in Figure 12 are the counterfactual paths corresponding to the case in which the average shocks start to hit the economy in 2007.Q4. The next blue dotted line corresponds to the case in which the random innovations start in 2008.Q3. The next two red dashed lines indicate the scenarios in which the average shocks start to hit the economy in 2008.Q4 and 2009.Q1. There are four outstanding messages from this exercise. First, macroeconomic conditions were deteriorating between 2007.Q4 and 2008.Q2. Second, even as late as 2008.Q2, the recession could have been averted if the subsequent shocks had been replaced with random draws from their historical distributions. Third, and more important, the Lehman shock in 2008.Q3 was key in the Great Recession. Note that the economy, in spite of avoiding adverse shocks immediately after Lehman's demise, never fully recovered in level and remained in a trajectory that coincides with the actual data at the end of 2011. Finally, the simulation suggests that conditions started to improve in 2009 and later, pushing the economy into recovery mode. This point can be seen by comparing the actual path with the counterfactual scenario in which randomization starts in 2009.Q1. In this scenario, the economy is buffeted by all the negative shocks realized in 2008 but misses favorable shocks (as suggested by the smoothed estimates) realized in subsequent years. Our simulation exercise suggests that in this scenario, the economy would have fallen into a downward spiral, as is consistent with the liquidity-frozen counterfactual exercise shown in Figure 9.

4.3 Avoiding the Great Recession

Identifying the transmission from the financial crisis to entrepreneurial activities as a critical force behind the Great Recession, our model suggests ex post that subsidizing R&D may be an effective way to cope with the crisis. We evaluate this idea in this section. This analysis is incidentally related to **?**, raising a sequence of thought-provoking questions including (i) if it is possible to undo what might now appear to be permanent changes, and (ii) what policy choices would give rise to alternative paths. We address these questions in a sense, using our estimated model as a laboratory.

We consider the subsity that brings down the price of R&D from 1 to $(1 - \tau_t^*)$, where $0 < \tau_t^* < 1$. The subsidy is financed by a lump-sum tax to households. We install this policy in our estimated model and simulate it with the actual realizations of the structural shocks. Figure 13 shows that, according to our estimated model, the economy could have avoided the Great

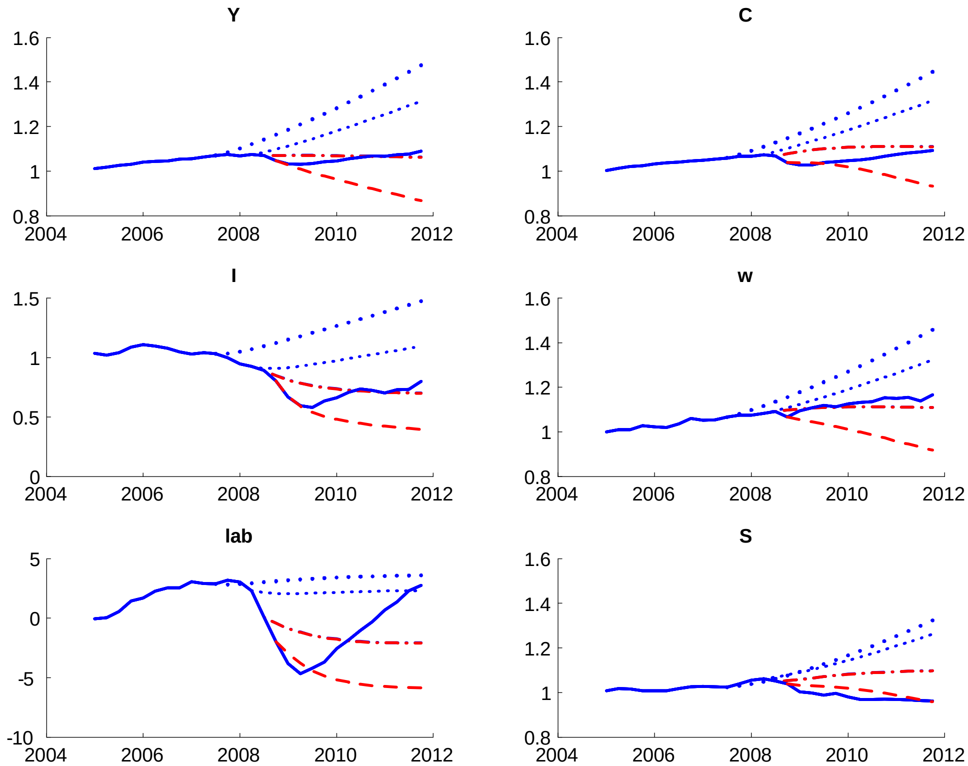

Figure 12: Trend Evolution During the Great Recession

Recession and could have grown at its steady state rate if entrepreneurs had received a subsidy as indicated in the top panel.[17] Notice that the subsidy is the mirror image of the liquidity shock (as shown in Figure 11), which indicates that the focus of this policy is canceling out the effect of the liquidity shock.

It is, however, a mistake to interpret the above policy as an easy solution to the crisis. On the contrary, the simulation presents a great challenge in implementing this policy in reality, because it is amazingly fast and astonishingly large. This point is clear by comparing it with an actual policy. In figure 14, we plot the fiscal stimulus in the American Recovery and Reinvestment Act (ARRA) of 2009,[18] as well as our hypothetical R&D subsidy measured in a percentage of GDP. Notice that the subsidy is larger and more "front-loaded" than the ARRA; in fact, about 4.3 percent of annual GDP is devoted to aid innovations in our simulation, and it happens in the first two critical years between the collapse of Lehman Brothers and the second quarter of 2010. In contrast, the cumulative size of the fiscal stimulus in the ARRA is about 1.6% of annual GDP up to the year 2012, and in addition, the stimulus gains momentum precisely at the time when the hypothetical R&D subsidy is tapering off. Given that the ARRA is an arguably hard-won political achievement, it is highly unlikely that the society quickly agrees upon implementing a

[17] R&D is taxed post 2010.Q2 in our model, but this is because financial conditions recovered according to our estimated path for liquidity (ϕ_t). If we lift the R&D tax, the economy grows at a rate above the historical trend.

[18] These are estimates of **?** exluding transfers.

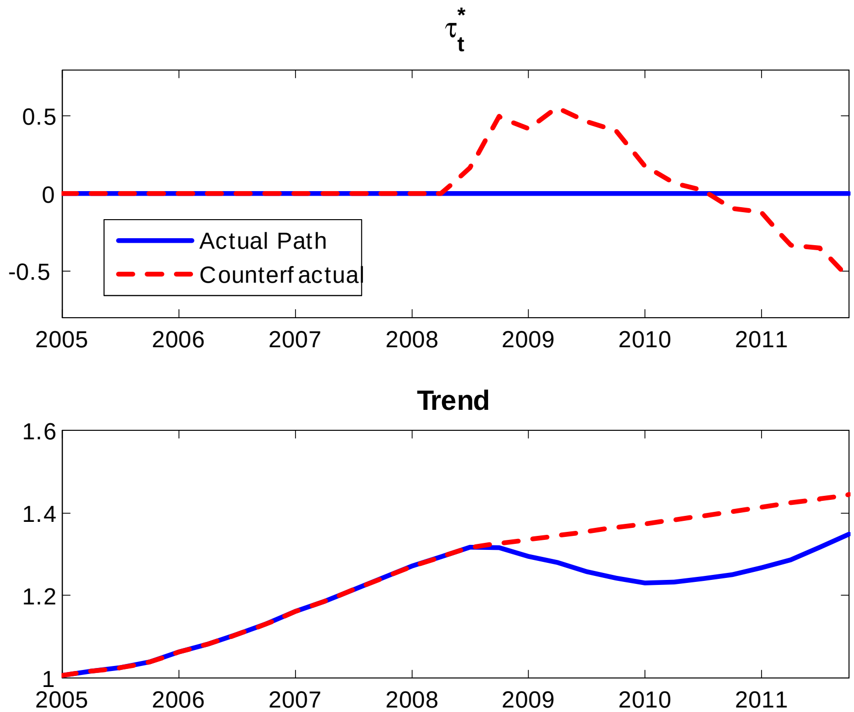

Figure 13: A Subsidy to Avoid the Great Recession

massive R&D subsidy we simulate. In other words, our analysis suggests that, if anything, only an unusual policy could undo the effects of the Great Recession.

4.4 RBC model with non-stationary productivity shock

So far we have looked at the recent U.S. macroeconomic history through the lens of our baseline model. Our finding is that the liquidity condition has consistently played important roles before, during, and after the Great Recession. In this section, we take a look at these episodes from a different perspective (i.e., through the lens of a standard RBC model). Specifically, we use a model that has productivity shock following a unit-root process and let it replace our endogenous growth mechanism. As we discussed in the introduction, such a model has the potential to account for the permanent downward shift in the economy's growth trend. Although this explanation is arguably mechanical, we believe that this perspective is worth exploring since, in terms of modeling, it is not only simpler, but it also more closely follows the real business cycle research tradition.

We make our model as simple and as clean as possible. We eliminate the research and development sector since the most basic RBC model is a one-sector model (e.g., Cooley and Prescott (1995)). For the similar reason, we abandon the intermediate goods sector too. However, we keep some important elements to facilitate comparison between our baseline model and our RBC-type alternative. Chief among them is the liquidity shock affecting investment decision. Specifically, following Del Negro, Eggertsson, Ferrero, and Kiyotaki (2011), Kiyotaki and Moore (2012), and

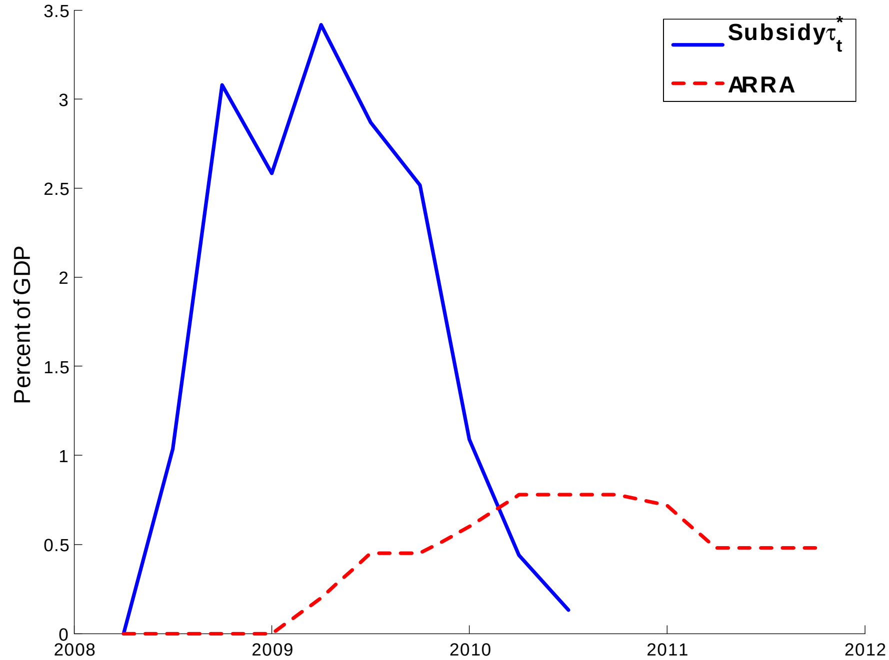

Figure 14: A Subsidy v.s. the American Recovery and Reinvestment Act (ARRA)

Shi (2012), we assume that investors are liquidity-constrained in a similar manner as are entrepreneurs in our baseline model. The model description is outlined in the appendix. Since there is no R&D sector in this version, there is one less shock (the one associated with the efficiency of R&D), which leads us to estimate the model with all observables but intangible investment. As with the baseline framework, the stochastic processes and the parameters related to adjustment costs in investment and capital utilization are the only objects that we estimate.

Figure 15 displays the estimated smoothed processes. At first, the estimated liquidity looks similar to the one obtained in our benchmark exercise. On closer look, however, we notice that the measure predicts that the worst part of the crisis happened in second half of 2009. Liquidity was relatively benign before and after the Lehman shock, according to this model. The depth of the liquidity crunch is substantially milder in the exogenous growth version (-0.25 percent below steady state versus -1.5 percent in the benchmark). This is hardly surprising since we are pushing productivity to be the key driver of the recession. Indeed, productivity bottomed in 2008.Q4. Interestingly, the time path for the discount factor points to households trying to de-leverage during the crisis (positive innovations mean a strong desire to save and consume less). This is in contrast to our benchmark model because the discount factor disturbance does not inflict permanent effects on the economy and hence does not cause a strong wealth effect.

Figure 16 compares the counterfactual paths when liquidity is frozen at its 2008.Q4 level in the exogenous growth model (red dotted line). The smaller role of the liquidity shock in this model is

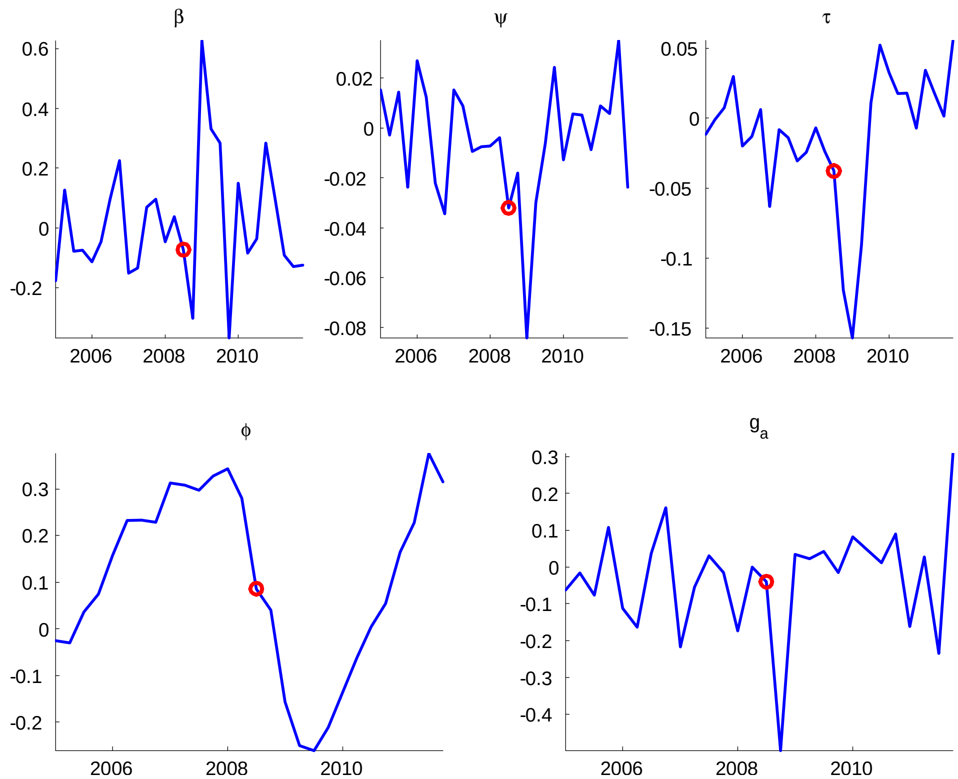

Figure 15: Smoothed Stochastic Processes in the Exogenous Growth Model

apparent from the fact that the counterfactual paths are almost indistinguishable from the actual paths (solid blue lines). As one might expect, a freeze in liquidity does not force the economy into a tailspin since liquidity leaves the economy's trend unscathed. For comparison, we also include the counterfactual from our baseline model (red dashed line), which indicates that in this model, contrary to the RBC model, the liquidity shock is crucially important to accounting for the Great Recession and the subsequent recovery.[19]

It is the productivity shock whose role the RBC model emphasizes the most. The following exercise identifies an extraordinarily large productivity drop in 2008.Q4 as the single most important cause of the Great Recession. We plot two counterfactual scenarios (Figure 17) in which productivity shocks are randomized starting from 2008.Q4 and 2009.Q1, respectively, and found that the recession could have been largely averted in the first scenario but not in the second. Note also that the smoothed stochastic processes indicate that the recovery in growth rate back to the pre-crisis level is mainly due to a reversal in productivity growth rather than financial conditions. Putting it another way, the RBC model attributes the shift in the U.S. trend to a combination of a large negative productivity drop and lackluster post-crisis productivity. To further underline the fact that the RBC model gives prominence to the productivity shock, we freeze this shock at

[19] There are two investment paths because, as explained in the main text, data on R&D spending is lumped together with investment in the model with exogenous growth to make the GDP series comparable across the two exercises.

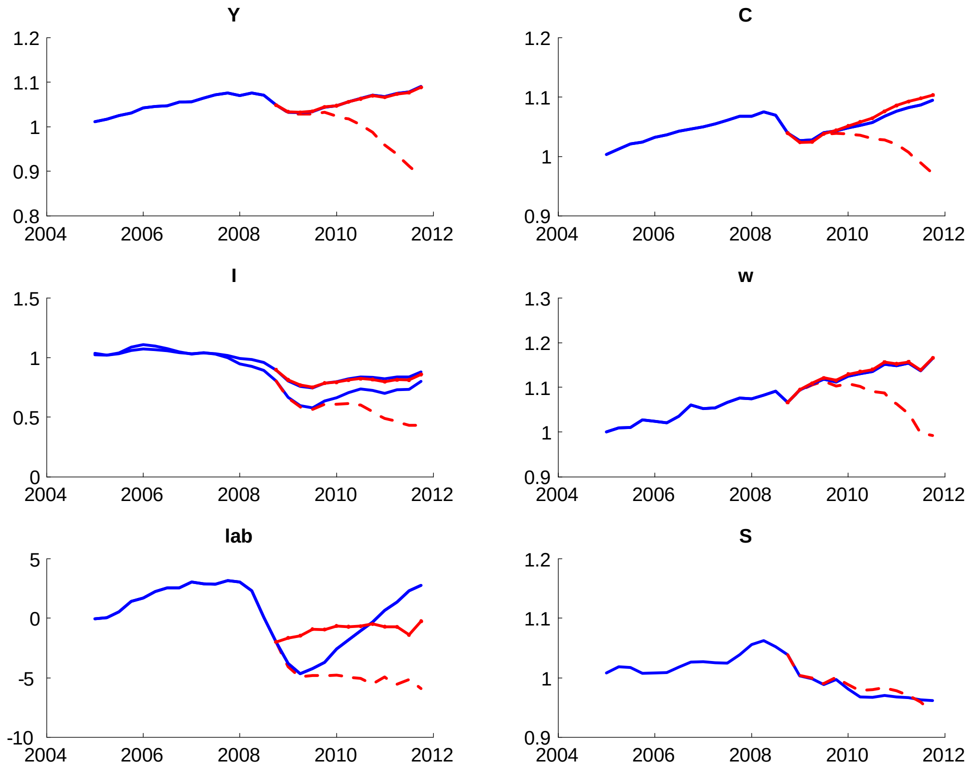

Figure 16: Liquidity Conditions Frozen in the RBC Model (red dotted line) and in the Endogenous Growth Model (red dashed line).

its 2008.Q4 level. As we see in Figure 18, the economy tailspins if the productivity growth rate is frozen (red dashed lines). It is interesting that the predicted trajectory is in fact similar to what our benchmark model predicts in the frozen liquidity counterfactual (Figure 9). In the RBC model, however, freezing the liquidity shock does not have much impact on the simulation (red dotted line).

5 A history of liquidity shocks

In our last exercise, we put the recent crisis in perspective. Figure 19 shows the smoothed paths for the liquidity process (ϕ_t) and the innovations buffeting it (the shaded areas correspond to recession periods as defined by the NBER). Several interesting points emerge from these figures. Our estimated paths imply that adverse liquidity conditions have been a common denominator over the past recessions. The 1975 and 1980/1982 contractions involved deep but short-lived drops in our measure of liquidity. Although the sizes of the innovations buffeting these crises and the Great Recession are comparable, it is clear that the economy quickly reverted to a state with better financial conditions in these episodes. In contrast, the post-Great Recession recovery has seen a milder improvement.

We view the drop in 1975 as the model's attempt to capture the sharp increase in oil prices.

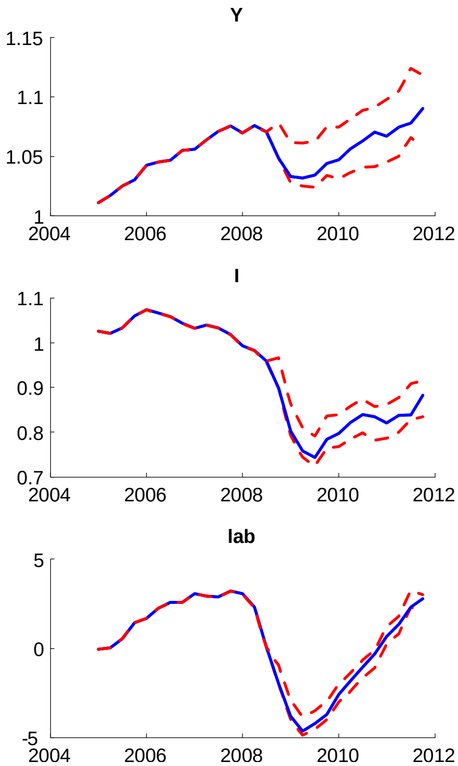

Figure 17: Counterfactual Paths in Exogenous Growth Model with Random Productivity Shocks

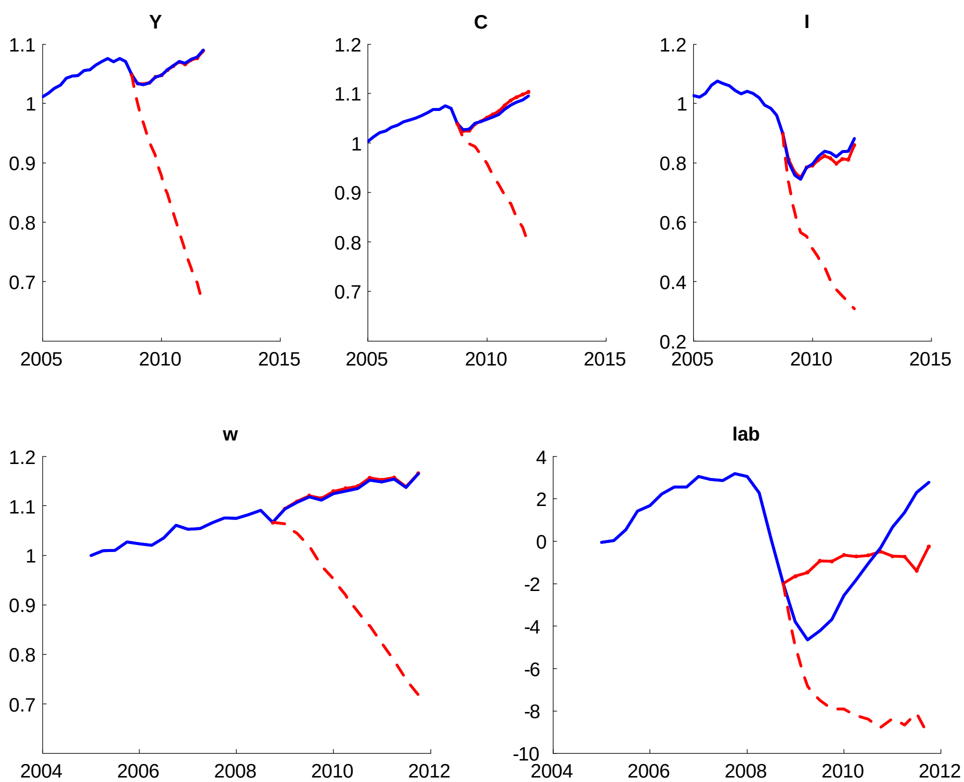

Figure 18: Productivity Growth Frozen (red dashed line) and Liquidity Conditions Frozen (red dotted line) in their 2008.Q4 Levels

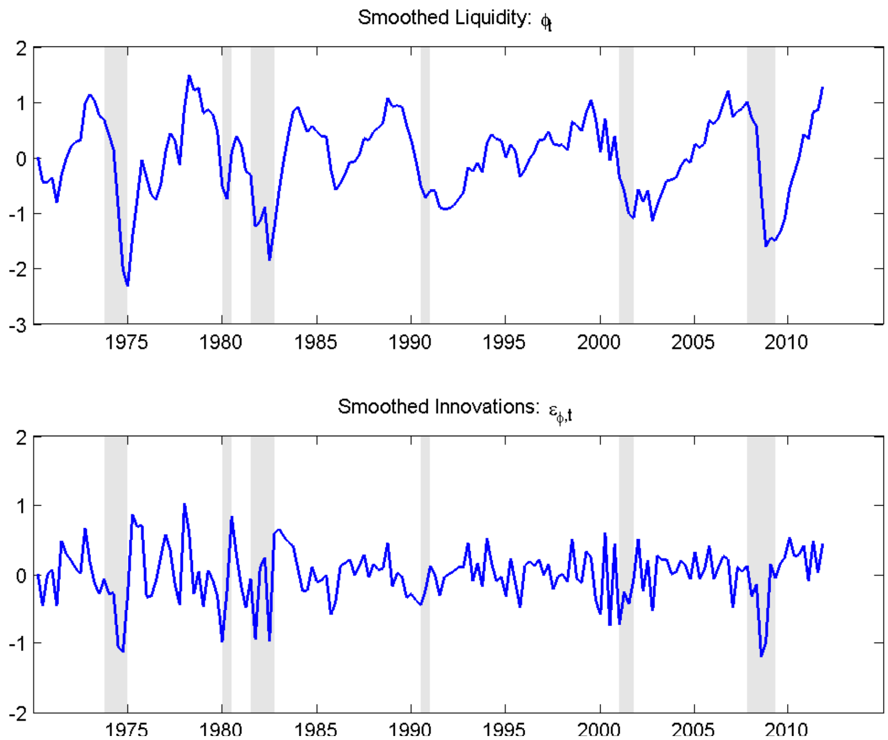

Figure 19: Time paths for liquidity and its innovations

Note that intermediate goods in our production function (17) enter in a similar way as in models with an oil sector. Note also that an expansion of product varieties reduces the price of intermediate goods composite relative to output, and vice versa. Hence, the model reads the adverse liquidity shock as a worsening of the production process to create intermediate goods. The decline of liquidity in 1980/1982 is such that, most likely, the model's interpretation of all the financial changes that arose at that time. Interestingly, our measure does pick up the Black Monday crash in 1987.

Consistent with the evidence reported in Brunnermeier and Pedersen (2009), liquidity fell during the first war in Iraq in the early 1990s. The estimates also reveal that the second half of the 1990s was a period of benign financial (or liquidity) conditions. Indeed, liquidity was above its historical average between 1994 and 2001. In our model, this implies ideal conditions for research and development and hence strong growth in the economy. In contrast, liquidity was very volatile during the 1970s and 1980s, which most likely resulted in the short periods of sustained growth that the economy experienced in those decades. A similar picture emerges during the last 10 years.

How likely was the Great Recession? Using the history of estimated liquidity shocks up to 2007.Q2, we find that a 95 percent probability set covers the region $(-0.97, 0.70)$. Although the shocks between 2007.Q3 and 2008.Q2 lied in that set, the innovation immediately after the failure of Lehman Brothers (-1.20) was a tail event. To see the sheer size of this shock, Figure 20 plots

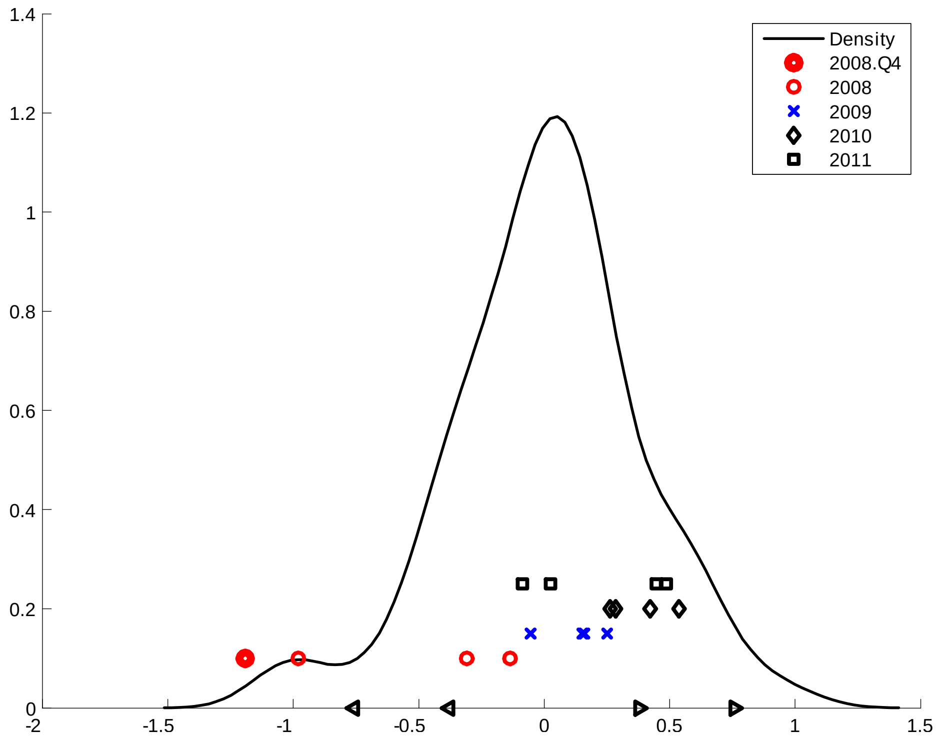

Figure 20: Density of Smoothed Liquidity Shocks $\varepsilon_{\phi,t}$

the density of liquidity shocks (2007.Q2 and before), the shock in 2008.Q4, and the sequence of shocks between 2008 and 2011 (the triangles on the horizontal axis represent standard deviation bands estimated from our model economy). Clearly, the liquidity shocks in 2008.Q3 and 2008.Q4 were extreme events well in excess of two-standard deviations. Moreover, the density also indicates that the distribution is skewed to the left (skewness = -0.29) and has a kurtosis of 3.7 (Fernández-Villaverde, Guerrón-Quintana, and Rubio-Ramírez (2014) provide a formal treatment of the role of asymmetric distributions in business cycles).

6 Conclusion

Adverse financial conditions during the recent recession seem to have played a critical role. Our model shows that financial shocks affecting the resaleability of equity is an example of the cross-winds the economy faced in 2008/2009. But illiquidity was far from being the solely financial malice. Default risk is another factor that most likely exacerbated the crisis. As a consequence, our model captures just one aspect of the Great Recession, and hence our results may well be a lower bound of the true impact that financial frictions had during the crisis.

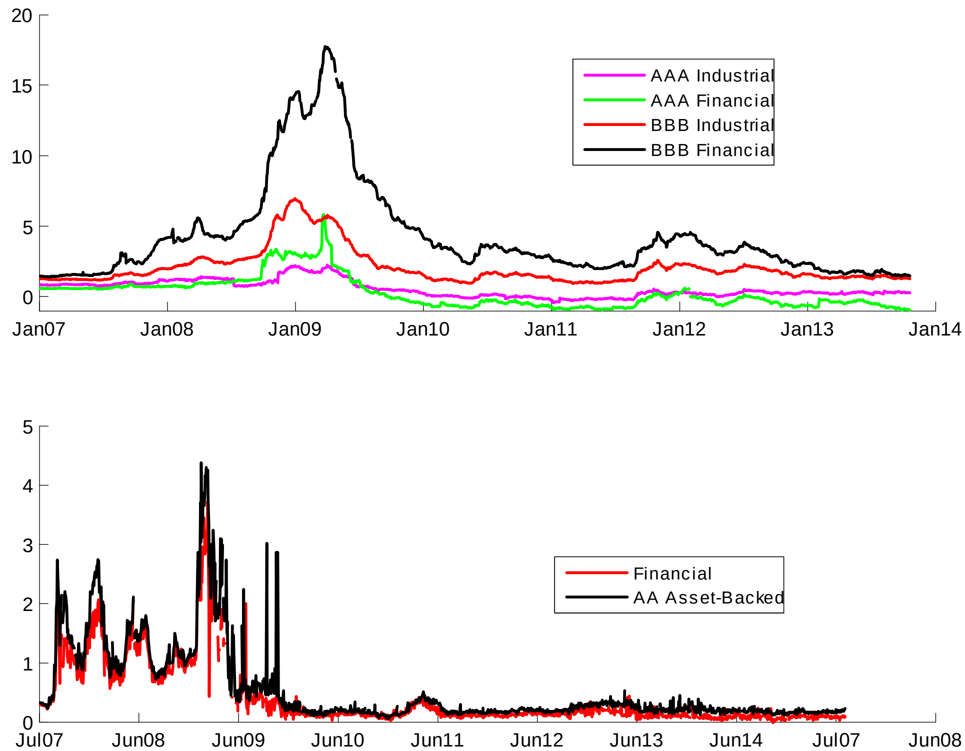

Figure 21: Credit Spreads and Corporate Bond Spreads

7 Appendix

7.1 Additional evidence on the Great Recession

Further evidence on harsh funding conditions comes from credit and corporate bond spreads. The upper panel of Figure 21 presents the corporate bond spreads (relative to 10-year Treasury bond yields at constant maturity). The bottom panel displays credit spreads computed as the difference between yields on 3-month (financial or asset-backed) commercial paper and 3-month constant maturity T-bills. These two measures point to higher spreads during the crisis. Investors moved away from commercial paper and into (the more liquid) Treasuries.

7.2 Liquidity constraints

This section shows that liquidity constraints (4) and (5) must be binding when conditions $1 < \vartheta_t p_{n,t} < 1/\theta$ hold. Equation (4) must be binding because otherwise, the household can increase the utility without violating any constraints by simultaneously increasing product developments and an entrepreneur's consumption from (s_t, c_t^e) to $(s_t + \Delta, c_t^e + (p_{n,t}\vartheta_t - 1)\,\Delta)$ as long as $\Delta > 0$ is sufficiently small. Equation (5) must be binding too because otherwise, the household can make (4) slack by letting entrepreneurs purchase equities and sell capital, i.e., changing from $\left(n_{t+1}^e, k_{t+1}^e\right)$ to $\left(n_{t+1}^e + (p_{k,t}/p_{n,t})\,\Delta, k_{t+1}^e - \Delta\right)$, and letting workers conduct the countertrading to offset the effects to the household's portfolio of assets at the end of the period. This strategy does not violate any constraints as long as $\Delta > 0$ is sufficiently small. But since the household can increase the utility if (4) is slack, this argument proves that (5) must be binding.

The argument so far relies only on $1 < \vartheta_t p_{n,t}$. The inequality from the other side $p_{n,t}\vartheta_t < 1/\theta$ is necessary for the household's problem to be properly formulated, because otherwise, revenues generated by product development are too large and therefore, an entrepreneur can self-finance any amount of product development costs by selling a fraction θ of new equities. In this case, there is no interior solution because changing the instruction to an entrepreneur from (s_t, c_t^e) to $(s_t + \Delta, c_t^e + (\theta p_{n,t}\vartheta_t - 1)\Delta)$ increases the equity holding and possibly entrepreneur's current consumption without violating any constraints, and Δ can be arbitrarily large.

7.3 Solving the household's problem

Given that liquidity constraints (4) and (5) hold with equalities, the household maximizes the value function (6) by choosing u_t, s_t, c_t^e, l_t, i_t, c_t^w, n_{t+1}^w, and k_{t+1}^w subject to

$$c_t^e + p_{n,t}\left[(1-\theta)\vartheta_t s_t - \phi_t(1-\delta_n)n_t\right] + p_{k,t}\left[-\phi_t(1-\delta(u_t))k_t\right] = \Pi_t n_t + R_t(u_t k_t) + (p_{n,t}\vartheta_t - 1)s_t - T_t, \tag{28}$$

$$c_t^w + i_t + p_{n,t}n_{t+1}^w + p_{k,t}k_{t+1}^w = \left[\Pi_t + p_{n,t}(1-\delta_n)\right]n_t + \left[R_t u_t + p_{k,t}(1-\delta(u_t))\right]k_t + W_t l_t - T_t, \tag{29}$$

$$n_{t+1} = \sigma_e\left[(1-\theta)\vartheta_t s_t + (1-\phi_t)(1-\delta_n)n_t\right] + \sigma_w n_{t+1}^w,$$

$$k_{t+1} = \sigma_e\left[(1-\phi_t)(1-\delta(u_t))k_t\right] + \sigma_w k_{t+1}^w + \left(1 - \Lambda\left(\frac{i_t}{i_{t-1}}\right)\right)(\sigma_w i_t).$$

First-order optimality conditions are

$$\beta_t E_t\left[\frac{\partial v_{t+1}}{\partial k_{t+1}}\right]\sigma_e(1-\phi_t)(-\delta'(u_t)) + \mu_t^e\left(p_{k,t}\phi_t(-\delta'(u_t)) + R_t\right) + \mu_t^w\left(R_t + p_{k,t}(-\delta'(u_t))\right) = 0, \tag{30}$$

$$\sigma_e\left(\frac{1}{c_t^e}\right) + \mu_t^e(-1) = 0, \tag{31}$$

$$\beta_t E_t\left[\frac{\partial v_{t+1}}{\partial n_{t+1}}\right]\sigma_e(1-\theta)\vartheta_t + \mu_t^e(-1 + \theta p_{n,t}\vartheta_t) = 0, \tag{32}$$

$$\sigma_w\left(\frac{1}{c_t^w}\right) + \mu_t^w(-1) = 0, \tag{33}$$

$$\sigma_w\left(-\psi_t l_t^\zeta\right) + \mu_t^w(W_t) = 0, \tag{34}$$

$$\beta_t E_t\left[\frac{\partial v_{t+1}}{\partial n_{t+1}}\right]\sigma_w + \mu_t^w(-p_{n,t}) = 0, \tag{35}$$

$$\beta_t E_t\left[\frac{\partial v_{t+1}}{\partial k_{t+1}}\right]\sigma_w + \mu_t^w(-p_{k,t}) = 0, \tag{36}$$

$$\beta_t E_t\left[\frac{\partial v_{t+1}}{\partial k_{t+1}}\right]\sigma_w\left(1-\Lambda\left(\frac{i_t}{i_{t-1}}\right)-\Lambda'\left(\frac{i_t}{i_{t-1}}\right)\frac{i_t}{i_{t-1}}\right)+\beta_t E_t\left[\frac{\partial v_{t+1}}{\partial i_t}\right]+\mu_t^w(-1)=0, \tag{37}$$

where μ_t^e and μ_t^w are the Lagrangian multipliers associated with (28) and (29), respectively. Envelope conditions are

$$\frac{\partial v_t}{\partial n_t}=\beta_t E_t\left[\frac{\partial v_{t+1}}{\partial n_{t+1}}\right]\sigma_e(1-\phi_t)(1-\delta_n)+\mu_t^e\left[\Pi_t+p_{n,t}(1-\delta_n)\phi_t\right]+\mu_t^w\left[\Pi_t+p_{n,t}(1-\delta_n)\right], \tag{38}$$

$$\frac{\partial v_t}{\partial k_t}=\beta_t E_t\left[\frac{\partial v_{t+1}}{\partial k_{t+1}}\right]\sigma_e(1-\phi_t)(1-\delta(u_t))+\mu_t^e\left[R_t u_t+p_{k,t}(1-\delta(u_t))\phi_t\right]+\mu_t^w\left[R_t u_t+p_{k,t}(1-\delta(u_t))\right], \tag{39}$$

$$\frac{\partial v_t}{\partial i_{t-1}}=\beta_t E_t\left[\frac{\partial v_{t+1}}{\partial k_{t+1}}\right]\sigma_w\Lambda'\left(\frac{i_t}{i_{t-1}}\right)\left(\frac{i_t}{i_{t-1}}\right)^2. \tag{40}$$

The optimality condition for labor supply (7) is derived by combining (33) and (34). The optimality condition for investment (8) is derived as follows. Combining (36) and (40), we find

$$\frac{\partial v_t}{\partial i_{t-1}}=\mu_t^w p_{k,t}\Lambda'\left(\frac{i_t}{i_{t-1}}\right)\left(\frac{i_t}{i_{t-1}}\right)^2. \tag{41}$$

Substituting (36) and (41) into (37), we obtain (8). Combining (33), (35), and (36) leads to (9) and (10). The optimality condition for R&D (11) is derived as follows. Combining (32) and (35), we find

$$\frac{\mu_t^e/\sigma_e}{\mu_t^w/\sigma_w}=\frac{p_{n,t}\vartheta_t(1-\theta)}{1-\theta p_{n,t}\vartheta_t}. \tag{42}$$

Combining (31), (33), and (42), we obtain (11).

The pricing equation for equity (14) is derived as follows. Combining (35) and (38), we find

$$\frac{\partial v_t}{\partial n_t}=\left(\frac{\mu_t^w}{\sigma_w}\right)\sigma_e(1-\phi_t)(1-\delta_n)p_{n,t}+\mu_t^e\left[\Pi_t+p_{n,t}(1-\delta_n)\phi_t\right]+\mu_t^w\left(\Pi_t+p_{n,t}(1-\delta_n)\right).$$

Substituting it into (35), we find

$$p_{n,t}=E_t\left[\begin{array}{c}\left(\beta_t\frac{\mu_{t+1}^w}{\mu_t^w}\right)\\ \left(\begin{array}{c}\sigma_e\left(1-\phi_{t+1}\right)(1-\delta_n)p_{n,t+1}\\ +\sigma_w\frac{\mu_{t+1}^e}{\mu_{t+1}^w}\left[\Pi_{t+1}+p_{n,t+1}(1-\delta_n)\phi_{t+1}\right]+\sigma_w\left(\Pi_{t+1}+p_{n,t+1}(1-\delta_n)\right)\end{array}\right)\end{array}\right].$$

Since $\sigma_w=1-\sigma_e$, we can rewrite the previous equation as

$$p_{n,t}=E_t\left[\begin{array}{c}\left(\beta_t\frac{\mu_{t+1}^w}{\mu_t^w}\right)\\ \left(\Pi_{t+1}+p_{n,t+1}(1-\delta_n)+\sigma_e\left(-1+\frac{\sigma_w}{\sigma_e}\frac{\mu_{t+1}^e}{\mu_{t+1}^w}\right)\left[\Pi_{t+1}+p_{n,t+1}(1-\delta_n)\phi_{t+1}\right]\right)\end{array}\right]. \tag{43}$$

Substituting (13) and (42) into (43), we obtain equation (14).

The pricing equation for capital (15) is derived as follows. Combining (36) and (39), we find

$$\frac{\partial v_t}{\partial k_t} = \left(\frac{\mu_t^w}{\sigma_w}\right) \sigma_e \left(1-\phi_t\right)\left(1-\delta\left(u_t\right)\right) p_{k,t} + \mu_t^e \left[R_t u_t + p_{k,t}\left(1-\delta\left(u_t\right)\right)\phi_t\right] + \mu_t^w \left(R_t u_t + p_{k,t}\left(1-\delta\left(u_t\right)\right)\right).$$

Substituting it into (36), we find

$$p_{k,t} = E_t \left[\begin{array}{c} \left(\beta_t \frac{\mu_{t+1}^w}{\mu_t^w}\right) \\ \left(\begin{array}{c} \sigma_e \left(1-\phi_{t+1}\right)\left(1-\delta\left(u_{t+1}\right)\right) p_{k,t+1} \\ +\sigma_w \frac{\mu_{t+1}^e}{\mu_{t+1}^w} \left[R_{t+1} u_{t+1} + p_{k,t+1}\left(1-\delta\left(u_{t+1}\right)\right)\phi_{t+1}\right] + \sigma_w \left(R_{t+1} u_{t+1} + p_{k,t+1}\left(1-\delta\left(u_{t+1}\right)\right)\right) \end{array} \right) \end{array} \right].$$

Since $\sigma_w = 1 - \sigma_e$, we can rewrite the previous equation as

$$p_{k,t} = E_t \left[\begin{array}{c} \left(\beta_t \frac{\mu_{t+1}^w}{\mu_t^w}\right) \\ \left(R_{t+1} u_{t+1} + p_{k,t+1}\left(1-\delta\left(u_{t+1}\right)\right) + \sigma_e \left(-1 + \frac{\sigma_w}{\sigma_e} \frac{\mu_{t+1}^e}{\mu_{t+1}^w}\right) \left[R_{t+1} u_{t+1} + p_{k,t+1}\left(1-\delta\left(u_{t+1}\right)\right)\phi_{t+1}\right]\right) \end{array} \right]. \quad (44)$$

Substituting (13) and (42) into (44), we obtain equation (15).

The optimality condition for the capacity utilization rate (16) is derived as follows. Combining (30) and (36), we find

$$\left(1-\phi_t\right)\left(-\delta'\left(u_t\right)\right) p_{k,t} + \frac{\sigma_w}{\sigma_e}\frac{\mu_t^e}{\mu_t^w}\left(R_t + p_{k,t}\phi_t\left(-\delta'\left(u_t\right)\right)\right) + \frac{\sigma_w}{\sigma_e}\left(R_t + p_{k,t}\left(-\delta'\left(u_t\right)\right)\right) = 0 \quad (45)$$

Substituting (13) and (42) into (45), we obtain equation (16).

7.4 Equilibrium properties

We find the relationship between the gross output, the value-added output, and the aggregate dividend as follows. The final goods firm's first-order condition (21) implies that

$$\xi Y_t = P_{G,t} G_t.$$

Since the revenue is decomposed into the costs and profits, we obtain

$$P_{G,t} G_t = N_t X_t + N_t \Pi_t.$$

Since intermediate goods firms charge a constant markup, the previous two equations imply that

$$N_t X_t = \frac{\xi}{\nu} Y_t,$$

$$N_t \Pi_t = \left(\frac{\nu - 1}{\nu}\right) \xi Y_t.$$

The relation between the gross output, the value-added output, and the aggregate dividend are therefore linear. Factor shares of rental income and labor income in the value-added output are constant, too.

We discuss the budget constraints in the equilibrium. Entrepreneur's budget constraint in the equilibrium is

$$c_t^e + p_{n,t}\left[(1-\theta)\vartheta_t s_t - \phi_t (1-\delta_n) N_t\right] - p_{k,t}\phi_t (1-\delta(u_t)) K_t = \Pi_t N_t + R_t (u_t K_t) + (p_{n,t}\vartheta_t - 1) s_t - T_t. \tag{46}$$

The worker's budget constraint in the equilibrium is

$$\begin{aligned} & c_t^w + i_t + \frac{p_{n,t}}{\sigma_w}\left(N_{t+1} - \sigma_e\left[(1-\theta)\vartheta_t s_t + (1-\phi_t)(1-\delta_n) N_t\right]\right) \\ & + \frac{p_{k,t}}{\sigma_w}(1-\delta(u_t))(1-\sigma_e(1-\phi_t)) K_t \\ = & \left[\Pi_t + p_{n,t}(1-\delta_n)\right] N_t + \left[R_t u_t + p_{k,t}(1-\delta(u_t))\right] K_t + W_t l_t - T_t. \end{aligned} \tag{47}$$

Adding (46) multiplied by σ_e and (47) multiplied by σ_w, we find

$$(\sigma_e c_t^e + \sigma_w c_t^w + \sigma_w i_t + \sigma_e s_t + T_t) - (R_t u_t K_t + W_t \sigma_w l_t + N_t \Pi_t) + p_{n,t}\left[N_{t+1} - (1-\delta_n) N_t - \vartheta_t (\sigma_e s_t)\right] = 0.$$

Combining this equation with (26) and (27), we find

$$p_{n,t}\left[N_{t+1} - (1-\delta_n) N_t - \vartheta_t (\sigma_e s_t)\right] = 0.$$

Since $p_{n,t}$ is always positive, we can drop the transition rule of N_t from the equilibrium conditions as long as we impose (26), (27), (46), and (47).

7.5 Model summary

The following 17 equations jointly determine the equilibrium dynamics of 17 endogenous variables, Y_t, Z_t, K_t, c_t^e, c_t^w, λ_t^e, s_t, $p_{n,t}$, Π_t, N_t, i_t, u_t, $p_{k,t}$, W_t, R_t, l_t, and ϑ_t:

$$Y_t = (u_t K_t)^\alpha (Z_t \sigma_w l_t)^{1-\alpha},$$

$$Z_t = \left(\bar{A}\right)(A_t N_t),$$

$$\psi_t l_t^\zeta = W_t \left(\frac{1}{c_t^w}\right),$$

$$W_t = (1-\xi)(1-\alpha)\frac{Y_t}{\sigma_w l_t},$$

$$\left(\frac{1}{c_t^e}\right) = (1+\lambda_t^e)\left(\frac{1}{c_t^w}\right),$$

$$\lambda_t^e = \frac{p_{n,t}\vartheta_t - 1}{1 - \theta p_{n,t}\vartheta_t},$$

$$p_{n,t} = E_t\left[\beta_t\left(\frac{c_t^w}{c_{t+1}^w}\right)\left(\Pi_{t+1} + p_{n,t+1}(1-\delta_n) + \sigma_e\lambda_{t+1}^e\left[\Pi_{t+1} + p_{n,t+1}(1-\delta_n)\phi_{t+1}\right]\right)\right],$$

$$\Pi_t = \left(\frac{\nu-1}{\nu}\right)\xi\frac{Y_t}{N_t},$$

$$R_t = (1-\xi)\alpha\frac{Y_t}{u_tK_t},$$

$$p_{k,t} = E_t\left[\beta_t\left(\frac{c_t^w}{c_{t+1}^w}\right)\left(R_{t+1}u_{t+1} + p_{k,t+1}(1-\delta(u_{t+1})) + \sigma_e\lambda_{t+1}^e\left[R_{t+1}u_{t+1} + p_{k,t+1}(1-\delta(u_{t+1}))\phi_{t+1}\right]\right)\right],$$

$$R_t + p_{k,t}(-\delta'(u_t)) + \sigma_e\lambda_t^e(R_t + p_{k,t}\phi_t(-\delta'(u_t))) = 0,$$

$$1 = p_{k,t}\left(1 - \Lambda\left(\frac{i_t}{i_{t-1}}\right) - \Lambda'\left(\frac{i_t}{i_{t-1}}\right)\frac{i_t}{i_{t-1}}\right) + E_t\left[\beta_t\left(\frac{c_t^w}{c_{t+1}^w}\right)p_{k,t+1}\Lambda'\left(\frac{i_{t+1}}{i_t}\right)\left(\frac{i_{t+1}}{i_t}\right)^2\right],$$

$$\begin{aligned}
& c_t^e + p_{n,t}\left[(1-\theta)\vartheta_t s_t - \phi_t(1-\delta_n)N_t\right] - p_{k,t}\phi_t(1-\delta(u_t))K_t \\
= \; & \Pi_t N_t + R_t(u_tK_t) + (p_{n,t}\vartheta_t - 1)s_t - \tau_t\left(1-\frac{\xi}{\nu}\right)Y_t,
\end{aligned}$$

$$\begin{aligned}
& c_t^w + i_t + \frac{p_{n,t}}{\sigma_w}\left(N_{t+1} - \sigma_e\left[(1-\theta)\vartheta_t s_t + (1-\phi_t)(1-\delta_n)N_t\right]\right) \\
& + \frac{p_{k,t}}{\sigma_w}(1-\delta(u_t))(1-\sigma_e(1-\phi_t))K_t \\
= \; & \left[\Pi_t + p_{n,t}(1-\delta_n)\right]N_t + \left[R_tu_t + p_{k,t}(1-\delta(u_t))\right]K_t + W_tl_t - \tau_t\left(1-\frac{\xi}{\nu}\right)Y_t,
\end{aligned}$$

$$\vartheta_t = \chi_t\left(\frac{\sigma_e s_t}{N_t}\right)^{\eta-1},$$

$$(1-\tau_t)\left(1-\frac{\xi}{\nu}\right)Y_t = \sigma_e c_t^e + \sigma_w c_t^w + \sigma_w i_t + \sigma_e s_t,$$

$$K_{t+1} = (1-\delta(u_t))K_t + \left(1-\Lambda\left(\frac{i_t}{i_{t-1}}\right)\right)(\sigma_w i_t).$$

Detrending them by N_t, we obtain a new system of equations with stationary variables. It is summarized by the following equations:

$$\hat{Y}_t = \left(u_t\hat{K}_t\right)^\alpha\left(\left(\bar{A}\right)(A_t)\left[\sigma_w l_t\right]\right)^{1-\alpha},$$

$$\psi_t l_t^\zeta = \frac{\hat{W}_t}{\hat{c}_t^w},$$

$$\hat{W}_t = (1-\xi)(1-\alpha)\frac{\hat{Y}_t}{\sigma_w l_t},$$

$$\frac{1}{\hat{c}_t^e} = (1+\lambda_t^e)\frac{1}{\hat{c}_t^w},$$

$$\lambda_t^e = \frac{p_{n,t}\vartheta_t - 1}{1-\theta p_{n,t}\vartheta_t},$$

$$p_{n,t} = E_t\left[\beta_t\left(\frac{1}{g_{t+1}}\frac{\hat{c}_t^w}{\hat{c}_{t+1}^w}\right)\left(\Pi_{t+1} + p_{n,t+1}(1-\delta_n) + \sigma_e\lambda_{t+1}^e\left[\Pi_{t+1} + p_{n,t+1}(1-\delta_n)\phi_{t+1}\right]\right)\right],$$

$$\Pi_t = \left(\frac{\nu-1}{\nu}\right)\xi\hat{Y}_t,$$

$$R_t = (1-\xi)\alpha\frac{\hat{Y}_t}{u_t\hat{K}_t},$$

$$p_{k,t} = E_t\left[\beta_t\left(\frac{1}{g_{t+1}}\frac{\hat{c}_t^w}{\hat{c}_{t+1}^w}\right)\left(R_{t+1}u_{t+1} + p_{k,t+1}(1-\delta(u_{t+1})) + \sigma_e\lambda_{t+1}^e\left[R_{t+1}u_{t+1} + p_{k,t+1}(1-\delta(u_{t+1}))\phi_{t+1}\right]\right)\right],$$

$$R_t + p_{k,t}(-\delta'(u_t)) + \sigma_e\lambda_t^e(R_t + p_{k,t}\phi_t(-\delta'(u_t))) = 0,$$

$$1 = p_{k,t}\left(1-\Lambda\left(g_t\frac{\hat{\imath}_t}{\hat{\imath}_{t-1}}\right) - \Lambda'\left(g_t\frac{\hat{\imath}_t}{\hat{\imath}_{t-1}}\right)\left(g_t\frac{\hat{\imath}_t}{\hat{\imath}_{t-1}}\right)\right) + E_t\left[\beta_t\left(\frac{1}{g_{t+1}}\frac{\hat{c}_t^w}{\hat{c}_{t+1}^w}\right)p_{k,t+1}\Lambda'\left(g_{t+1}\frac{\hat{\imath}_{t+1}}{\hat{\imath}_t}\right)\left(g_{t+1}\frac{\hat{\imath}_{t+1}}{\hat{\imath}_t}\right)^2\right],$$

$$\begin{aligned}
&\hat{c}_t^e + p_{n,t}\left[(1-\theta)\vartheta_t\hat{s}_t - \phi_t(1-\delta_n)\right] - p_{k,t}\phi_t(1-\delta(u_t))\hat{K}_t \\
=\;& \Pi_t + R_t\left(u_t\hat{K}_t\right) + (p_{n,t}\vartheta_t - 1)\hat{s}_t - \tau_t\left(1-\frac{\xi}{\nu}\right)\hat{Y}_t,
\end{aligned}$$

$$\begin{aligned}
&\hat{c}_t^w + \hat{\imath}_t + \frac{p_{n,t}}{\sigma_w}\left(g_{t+1} - \sigma_e\left[(1-\theta)\vartheta_t\hat{s}_t + (1-\phi_t)(1-\delta_n)\right]\right) \\
&+\frac{p_{k,t}}{\sigma_w}(1-\delta(u_t))(1-\sigma_e(1-\phi_t))\hat{K}_t \\
=\;& \Pi_t + p_{n,t}(1-\delta_n) + \left[R_t u_t + p_{k,t}(1-\delta(u_t))\right]\hat{K}_t + \hat{W}_t l_t - \tau_t\left(1-\frac{\xi}{\nu}\right)\hat{Y}_t,
\end{aligned}$$

$$\vartheta_t = \chi_t(\sigma_e\hat{s}_t)^{\eta-1},$$

$$(1-\tau_t)\left(1-\frac{\xi}{\nu}\right)\hat{Y}_t = \sigma_e\hat{c}_t^e + \sigma_w\hat{c}_t^w + \sigma_w\hat{\imath}_t + \sigma_e\hat{s}_t,$$

$$g_{t+1}\hat{K}_{t+1} = (1-\delta(u_t))\hat{K}_t + \left(1-\Lambda\left(g_t\frac{\hat{\imath}_t}{\hat{\imath}_{t-1}}\right)\right)(\sigma_w\hat{\imath}_t).$$

Hat variables denote the original variables divided by N_t, i.e., $\hat{Y}_t = Y_t/N_t$, and so on, and $g_{t+1} = N_{t+1}/N_t$.

7.6 Steady state

This section briefly describes how to find the steady state and the calibrated parameter values. Both the steady state neutral productivity shock A and the steady state labor disutility shock ψ are set so that both the steady state gross output $\hat{Y}$ and the steady state labor l are equal to 1. The steady state sector-specific productivity shock χ, the steady state subjective discount factor β, and the steady state capital depreciation rate δ_k are set so that the steady state growth rate g, the steady state R&D share $\sigma_e \hat{s}/\hat{\mathcal{Y}}$, and the steady state investment share $\sigma_w \hat{\imath}/\hat{\mathcal{Y}}$ match their empirical counterparts. The steady state values of endogenous variables are sequentially found as follows:

$$\Pi = \frac{\Pi}{\hat{Y}} = \underbrace{\left(\frac{\nu - 1}{\nu}\right)\xi}_{\text{known}},$$

$$\sigma_e \hat{s} = \frac{\sigma_e \hat{s}}{\hat{Y}} = \underbrace{\left(1 - \frac{\xi}{\nu}\right)\left(\frac{\sigma_e \hat{s}}{\hat{\mathcal{Y}}}\right)}_{\text{known}},$$

$$\hat{s} = \underbrace{(\sigma_e \hat{s})\left(\frac{1}{\sigma_e}\right)}_{\text{known}},$$

$$\vartheta = \underbrace{\frac{g - 1 + \delta_n}{\sigma_e \hat{s}}}_{\text{known}},$$

$$p_k = 1,$$

$$\hat{W} = (1 - \xi)(1 - \alpha)\frac{\hat{Y}}{\sigma_w l} = \underbrace{(1 - \xi)(1 - \alpha)\frac{1}{\sigma_w}}_{\text{known}},$$

$$\sigma_w \hat{\imath} = \frac{\sigma_w \hat{\imath}}{Y} = \underbrace{\left(1 - \frac{\xi}{\nu}\right)\left(\frac{\sigma_w \hat{\imath}}{\hat{\mathcal{Y}}}\right)}_{\text{known}},$$

$$\hat{\imath} = \underbrace{(\sigma_w \hat{\imath})\left(\frac{1}{\sigma_w}\right)}_{\text{known}},$$

$$Ru = (1 - \xi)\alpha\frac{\hat{Y}}{\hat{K}} = (1 - \xi)\alpha\frac{1}{\hat{K}},$$

$$\hat{K} = \frac{\sigma_w \hat{\imath}}{g - 1 + \delta_k}.$$

The following equations solve $\hat{c}^e$, $\hat{c}^w$, λ^e, p_n, β, and δ_k.

$$\hat{c}^w = (1 + \lambda^e)\hat{c}^e,$$

$$\lambda^e = \frac{(p_n)(\vartheta) - 1}{1 - \theta (p_n)(\vartheta)},$$

$$p_n = \left(\frac{\beta}{g}\right)\left(\Pi + p_n (1 - \delta_n) + \sigma_e \lambda^e \left[\Pi + p_n (1 - \delta_n)\phi\right]\right),$$

$$1 = \left(\frac{\beta}{g}\right)\left((1-\xi)\alpha \frac{g - 1 + \delta_k}{\sigma_w \hat{\imath}} + 1 - \delta_k + \sigma_e \lambda^e \left[(1-\xi)\alpha \frac{g - 1 + \delta_k}{\sigma_w \hat{\imath}} + (1 - \delta_k)\phi\right]\right),$$

$$\hat{c}^e + p_n \left[(1-\theta)(\vartheta)(\hat{s}) - \phi(1 - \delta_n)\right] - \phi (1 - \delta_k) \frac{\sigma_w \hat{\imath}}{g - 1 + \delta_k} = \Pi + (1-\xi)\alpha + (p_n(\vartheta) - 1)(\hat{s}) - \tau \left(1 - \frac{\xi}{\nu}\right),$$

$$(1 - \tau)\left(1 - \frac{\xi}{\nu}\right) = \sigma_e \hat{c}^e + \sigma_w \hat{c}^w + \sigma_w \hat{\imath} + \sigma_e \hat{s}.$$

Other steady state values are found by

$$\hat{K} = \underbrace{\frac{\sigma_w \hat{\imath}}{g - 1 + \delta_k}}_{\text{known}},$$

$$R = \underbrace{(1-\xi)\alpha \frac{1}{u}\frac{1}{\hat{K}}}_{\text{known}},$$

$$\delta'(u)\, u = \underbrace{\left(\frac{1 + \sigma_e \lambda^e}{1 + \sigma_e \lambda^e \phi}\right) Ru}_{\text{known}}.$$

The parameter values are found by

$$A = \underbrace{\left(u\hat{K}\right)^{\frac{\alpha}{\alpha - 1}} \left(\bar{A}\sigma_w\right)^{-1}}_{\text{known}},$$

$$\psi = \underbrace{\frac{\hat{W}}{\hat{c}^w}}_{\text{known}},$$

$$\chi = \underbrace{\frac{\vartheta}{(\sigma_e \hat{s})^{\eta - 1}}}_{\text{known}}.$$

7.7 RBC model with non-stationary productivity shock

We consider a large household that has a unit measure of members. A member will be an investor with probability $\sigma_i \in [0, 1]$ and a worker with probability $\sigma_w \in [0, 1]$. They satisfy $\sigma_i + \sigma_w = 1$. These shocks are *iid* among the members and across time.

A period is divided into four stages: household's decisions, production, investment, and consumption. In the stage of household's decisions, all members of a household are together to pool their assets, i.e., k_t units of capital. Aggregate shocks to exogenous state variables are realized. The capacity utilization rate u_t is decided and is applied to all the capital the household possesses. Because the members in the household are identical in this stage, the household evenly divides the assets among the members. The head of the household also gives contingency plans to each member, saying that if one becomes an investor, he or she invests i_t units of consumption goods, consumes c_t^i units of consumption goods, and makes necessary trades in the capital market so that he or she returns to the household with k_{t+1}^i units of capital, and if one becomes a worker, he or she supplies l_t units of labor, consumes c_t^w units of consumption goods, and makes necessary trades in the capital market so that he or she returns to the household with k_{t+1}^w units of capital. After receiving these instructions, the members go to the market and will remain separated from each other for the remaining of the period.

At the beginning of the production stage, each member receives the shock whose realization determines whether the individual is an investor or a worker. Competitive firms produce final consumption goods from capital service and labor service. After production, a worker receives wage income, and an individual receives compensation for capital service. The government collects a uniform, lump-sum tax T_t from each member. Then, a fraction $\delta(u_t)$ of capital depreciates.

The third stage in the period is the investment stage. The goods market and the capital market are open, and investors seek financing and undertake investment as instructed by the household. We assume that an investor can transform i_t units of consumption goods into the following units of capital goods

$$\left(1 - \Lambda\left(\frac{i_t}{i_{t-1}^*}\right)\right) i_t$$

where $\Lambda(\cdot)$ is the capital adjustment costs given by

$$\Lambda\left(\frac{i_t}{i_{t-1}^*}\right) = \frac{\bar{\Lambda}}{2}\left(g - \frac{i_t}{i_{t-1}^*}\right)^2$$

i_{t-1}^* is the cross-sectional average investment level in the economy, which an individual household takes as given, and g is the steady state growth rate of the technology level.[20] Individuals trade assets as instructed earlier by their households. In the consumption stage, both a worker and an investor consume.

The instructions have to satisfy a set of constraints. First, the instructions to an investor has

[20] Notice that the investment adjustment cost function is formally similar to the one in the baseline model since $i_{t-1}^* = i_{t-1}$ holds in equilibrium, but is different from it because the household in the current model does not internalize the effects of current investment on future investment adjustment costs. This is for analytical convenience; namely, the liquidity constraint and dynamic investment adjustment costs are hard to analyze together. Preceding studies (Del Negro, Eggertsson, Ferrero, and Kiyotaki (2011) and Shi (2012)) make similar assumptions so that investment adjustment costs only depend on the current investment level.

to satisfy the intra-period budget constraint:

$$\underbrace{c_t^i + i_t + \underbrace{p_{k,t}k_{t+1}^i}_{\text{gross equity purchases}}}_{\text{gross expenditure}} = \underbrace{\underbrace{R_t\left(u_t k_t\right)}_{\text{rental}} + p_{k,t}\underbrace{\left[\left(1-\delta\left(u_t\right)\right)k_t + \left(1-\Lambda\left(\frac{i_t}{i_{t-1}^*}\right)\right)i_t\right]}_{\text{resale value + value of new capital}} - T_t}_{\text{gross after-tax income}}. \tag{48}$$

The interpretation is analogous to the one in our benchmark model. A similar constraint applies to a worker:

$$c_t^w + p_{k,t}k_{t+1}^w = R_t\left(u_t k_t\right) + p_{k,t}\left(1-\delta\left(u_t\right)\right)k_t + W_t l_t - T_t. \tag{49}$$

There are other, crucial constraints on the trading of assets. That is, an investor can sell at most a fraction θ of newly installed capital but has to retain the rest by herself. In addition, she can sell a fraction ϕ_t of existing capital to others in the asset markets, but has to retain the rest by herself. Effectively, these constraints introduce a lower bound to the capital holding at the end of the period:

$$k_{t+1}^i \geq \underbrace{\left(1-\theta\right)\left(1-\Lambda\left(\frac{i_t}{i_{t-1}^*}\right)\right)i_t}_{\text{newly installed capital required to retain}} + \underbrace{\left(1-\phi_t\right)\left(1-\delta\left(u_t\right)\right)k_t}_{\text{existing capital required to retain}}. \tag{50}$$

A similar constraint applies to workers, i.e., $k_{t+1}^w \geq \left(1-\phi_t\right)\left(1-\delta\left(u_t\right)\right)k_t$, but we omit it because it does not bind in the equilibrium. There are non-negativity constraints for u_t, c_t^i, l_t, i_t, c_t^w, and k_{t+1}^w, but we omit them too, for the same reason.

The head of the household chooses the instructions to its members to maximize the value function defined as

$$v\left(k_t;\Gamma_t,\Theta_t\right) = \max\left\{\sigma_i \log\left(c_t^i\right) + \sigma_w\left[\log\left(c_t^w\right) - \psi_t\frac{l_t^{1+\zeta}}{1+\zeta}\right] + \beta_t E_t\left[v\left(k_{t+1};\Gamma_{t+1},\Theta_{t+1}\right)\right]\right\} \tag{51}$$

subject to (48), (49), (50), and

$$k_{t+1} = \sigma_i k_{t+1}^i + \sigma_w k_{t+1}^w \tag{52}$$

Γ_t is the vector of endogenous, aggregate state variables, i.e., $\Gamma_t = \left(K_t, i_{t-1}^*\right)$, where K_t is the capital stock in the economy. Θ_t is the vector of exogenous state variables.

We will restrict our attention to the case in which the inequality $p_{k,t} > p_{i,t}$ always holds in the equilibrium, where $p_{i,t}$ is the shadow price of newly installed capital defined as

$$p_{i,t} \equiv \left(1-\Lambda\left(\frac{i_t}{i_{t-1}^*}\right) - \Lambda'\left(\frac{i_t}{i_{t-1}^*}\right)\frac{i_t}{i_{t-1}^*}\right)^{-1}. \tag{53}$$

The liquidity constraint (50) must be binding because otherwise the household can increase the utility without violating any constraints by marginally increasing investment, which creates $1/p_{i,t}$ units of new capital, selling it in the capital market, and increasing an investor's consumption by $(p_{k,t}/p_{i,t}-1)$ units.

The representative final goods producing firm uses capital service KS_t and labor L_t to produce the final (consumption) good according to the production technology

$$Y_t = (KS_t)^{\alpha}(A_t L_t)^{1-\alpha}.$$

A_t is the productivity shock following a non-stationary process:

$$A_t = e^{a_t},$$

$$\Delta a_t = \log(g) + \rho_a \Delta a_{t-1} + \varepsilon_{a,t}.$$

The firm is competitive, maximizing profits defined as

$$Y_t - R_t(KS_t) - W_t L_t.$$

The government spends a fraction τ_t of the output Y_t. We assume that the government keeps the balanced budget:

$$\tau_t Y_t = T_t.$$

The competitive equilibrium is defined in a standard way. Market clearing conditions for production factors are

$$KS_t = u_t K_t,$$

$$L_t = \sigma_w l_t.$$

The goods market clearing condition is

$$(1-\tau_t) Y_t = \sigma_i c_t^i + \sigma_w c_t^w + \sigma_i i_t. \tag{54}$$

Asset market clearing conditions are

$$K_t = k_t,$$

at the beginning of the period and

$$K_{t+1} = k_{t+1} = \sigma_i k_{t+1}^i + \sigma_w k_{t+1}^w,$$

at the end of the period. $i_{t-1}^* = i_{t-1}$ holds in the equilibrium and therefore the aggregate capital stock K_t evolves as

$$K_{t+1} = (1-\delta(u_t)) K_t + \left(1 - \Lambda\left(\frac{i_t}{i_{t-1}}\right)\right)(\sigma_i i_t).$$

The derivations of the equilibrium conditions are analogous to those in the benchmark model. The following equations summarize the model economy:

$$Y_t = (u_t K_t)^{\alpha} \left(e_t^{a_t} \sigma_w l_t\right)^{1-\alpha},$$

$$\psi_t l_t^{\zeta} = W_t \frac{1}{c_t^w},$$

$$W_t = (1-\alpha) \frac{Y_t}{\sigma_w l_t},$$

$$\frac{1}{c_t^i} = (1+\lambda_t) \frac{1}{c_t^w},$$

$$\lambda_t = \frac{(p_{k,t}/p_{i,t}) - 1}{1 - \theta \left(p_{k,t}/p_{i,t}\right)},$$

$$R_t = \alpha \frac{Y_t}{u_t K_t},$$

$$1 = p_{i,t} \left(1 - \Lambda\left(\frac{i_t}{i_{t-1}}\right) - \Lambda'\left(\frac{i_t}{i_{t-1}}\right) \frac{i_t}{i_{t-1}}\right),$$

$$p_{k,t} = E_t \left[\left(\beta_t \frac{c_t^w}{c_{t+1}^w}\right) \left(R_{t+1} u_{t+1} + p_{k,t+1} \left(1 - \delta\left(u_{t+1}\right)\right) + \sigma_i \lambda_{t+1} \left[R_{t+1} u_{t+1} + p_{k,t+1} \left(1 - \delta\left(u_{t+1}\right)\right) \phi_{t+1}\right]\right)\right],$$

$$R_t + p_{k,t} \left(-\delta'\left(u_t\right)\right) + \sigma_i \lambda_t \left(R_t + p_{k,t} \phi_t \left(-\delta'\left(u_t\right)\right)\right) = 0,$$

$$c_t^i + i_t + p_{k,t} \left[-\theta \left(1 - \Lambda\left(\frac{i_t}{i_{t-1}}\right)\right) i_t - \phi_t \left(1 - \delta\left(u_t\right)\right) K_t\right] = R_t \left(u_t K_t\right) - \tau_t Y_t$$

$$\begin{aligned} & c_t^w + \frac{p_{k,t}}{\sigma_w} \left(K_{t+1} - \sigma_i \left[(1-\theta) \left(1 - \Lambda\left(\frac{i_t}{i_{t-1}}\right)\right) i_t + (1 - \phi_t) \left(1 - \delta\left(u_t\right)\right) K_t\right]\right) \\ = \; & \left[R_t u_t + p_{k,t} \left(1 - \delta\left(u_t\right)\right)\right] K_t + W_t l_t - \tau_t Y_t, \end{aligned}$$

$$(1 - \tau_t) Y_t = \sigma_i c_t^i + \sigma_w c_t^w + \sigma_i i_t.$$

The 12 equations jointly determine the equilibrium dynamics of 12 endogenous variables, Y_t, K_t, c_t^i, c_t^w, λ_t, i_t, u_t, $p_{k,t}$, $p_{i,t}$, W_t, R_t, and l_t.

7.8 R&D subsidy

We will consider a subsidy to R&D. The household maximizes the value function (6) subject to (1), (3), (4), (5), and

$$c_t^e + (1 - \tau_t^*) s_t + p_{n,t} n_{t+1}^e + p_{k,t} k_{t+1}^e = \Pi_t n_t + R_t (u_t k_t) + p_{n,t} (1 - \delta_n) n_t + p_{k,t} (1 - \delta(u_t)) k_t + p_{n,t} \vartheta_t s_t - T_t$$

$$n_{t+1} = \sigma_e n_{t+1}^e + \sigma_w n_{t+1}^w$$

where $\tau_t^* \geq 0$ is the subsidy. We will restrict our attention to the case in which $1 - \tau_t^* < p_{n,t}\vartheta_t < (1 - \tau_t^*) / \theta$ always hold. The entrepreneur's liquidity constraints bind at the optimum as long as these constraints hold. First order conditions are

$$\beta_t E_t \left[\frac{\partial v_{t+1}}{\partial k_{t+1}}\right] \sigma_e (1 - \phi_t) (-\delta'(u_t)) + \mu_t^e (p_{k,t}\phi_t (-\delta'(u_t)) + R_t) + \mu_t^w (R_t + p_{k,t} (-\delta'(u_t))) = 0,$$

$$\sigma_e \left(\frac{1}{c_t^e}\right) + \mu_t^e (-1) = 0, \tag{55}$$

$$\beta_t E_t \left[\frac{\partial v_{t+1}}{\partial n_{t+1}}\right] \sigma_e (1 - \theta) \vartheta_t + \mu_t^e [-(1 - \tau_t^*) + \theta p_{n,t}\vartheta_t] = 0, \tag{56}$$

$$\sigma_w \left(\frac{1}{c_t^w}\right) + \mu_t^w (-1) = 0, \tag{57}$$

$$\sigma_w \left(-\psi_t l_t^\zeta\right) + \mu_t^w (W_t) = 0, \tag{58}$$

$$\beta_t E_t \left[\frac{\partial v_{t+1}}{\partial n_{t+1}}\right] \sigma_w + \mu_t^w (-p_{n,t}) = 0, \tag{59}$$

$$\beta_t E_t \left[\frac{\partial v_{t+1}}{\partial k_{t+1}}\right] \sigma_w + \mu_t^w (-p_{k,t}) = 0,$$

$$\beta_t E_t \left[\frac{\partial v_{t+1}}{\partial k_{t+1}}\right] \sigma_w \left(1 - \Lambda\left(\frac{i_t}{i_{t-1}}\right) - \Lambda'\left(\frac{i_t}{i_{t-1}}\right)\frac{i_t}{i_{t-1}}\right) + \beta_t E_t \left[\frac{\partial v_{t+1}}{\partial i_t}\right] + \mu_t^w (-1) = 0.$$

Envelope conditions are

$$\frac{\partial v_t}{\partial n_t} = \beta_t E_t \left[\frac{\partial v_{t+1}}{\partial n_{t+1}}\right] \sigma_e (1 - \phi_t)(1 - \delta_n) + \mu_t^e [\Pi_t + p_{n,t} (1 - \delta_n) \phi_t] + \mu_t^w [\Pi_t + p_{n,t} (1 - \delta_n)],$$

$$\frac{\partial v_t}{\partial k_t} = \beta_t E_t \left[\frac{\partial v_{t+1}}{\partial k_{t+1}}\right] \sigma_e (1 - \phi_t)(1 - \delta(u_t)) + \mu_t^e [R_t u_t + p_{k,t} (1 - \delta(u_t)) \phi_t] + \mu_t^w [R_t u_t + p_{k,t} (1 - \delta(u_t))],$$

$$\frac{\partial v_t}{\partial i_{t-1}} = \beta_t E_t \left[\frac{\partial v_{t+1}}{\partial k_{t+1}}\right] \sigma_w \Lambda'\left(\frac{i_t}{i_{t-1}}\right)\left(\frac{i_t}{i_{t-1}}\right)^2.$$

The only difference from the benchmark model is the first order condition with respect to s_t (equation (56)). This difference affects liquidity services. Specifically, combining (56) and (59), we obtain

$$\frac{\mu_t^e/\sigma_e}{\mu_t^w/\sigma_w} = \frac{p_{n,t}(1 - \theta)\vartheta_t}{(1 - \tau_t^*) - \theta p_{n,t}\vartheta_t}. \tag{60}$$

Combining (55), (57), and (60), we obtain

$$\frac{1}{c_t^e} = (1 + \lambda_t^*) \frac{1}{c_t^w}$$

where

$$\lambda_t^* = \frac{p_{n,t}\vartheta_t - (1-\tau_t^*)}{(1-\tau_t^*) - \theta p_{n,t}\vartheta_t}.$$

As in the benchmark model, liquidity services λ_t^* measure the relative importance to the household between the incremental resource given to an entrepreneur and the incremental resource given to a worker. λ_t^* increases with the subsidy τ_t^* because the subsidy increases the household's incentive to conduct R&D in an otherwise same environment. Other optimality conditions are derived analogously:

$$\psi_t l_t^\zeta = W_t \left(\frac{1}{c_t^w}\right),$$

$$1 = p_{k,t}\left(1 - \Lambda\left(\frac{i_t}{i_{t-1}}\right) - \Lambda'\left(\frac{i_t}{i_{t-1}}\right)\frac{i_t}{i_{t-1}}\right) + E_t\left[\left(\beta_t \frac{c_t^w}{c_{t+1}^w}\right) p_{k,t+1}\Lambda'\left(\frac{i_{t+1}}{i_t}\right)\left(\frac{i_{t+1}}{i_t}\right)^2\right],$$

$$p_{n,t} = E_t\left[\left(\beta_t \frac{c_t^w}{c_{t+1}^w}\right)\left(\Pi_{t+1} + p_{n,t+1}(1-\delta_n) + \sigma_e\lambda_{t+1}^*\left[\Pi_{t+1} + p_{n,t+1}(1-\delta_n)\phi_{t+1}\right]\right)\right],$$

$$p_{k,t} = E_t\left[\left(\beta_t \frac{c_t^w}{c_{t+1}^w}\right)\left(R_{t+1}u_{t+1} + p_{k,t+1}(1-\delta(u_{t+1})) + \sigma_e\lambda_{t+1}^*\left[R_{t+1}u_{t+1} + p_{k,t+1}(1-\delta(u_{t+1}))\phi_{t+1}\right]\right)\right],$$

$$R_t + p_{k,t}(-\delta'(u_t)) + \sigma_e\lambda_t^*(R_t + p_{k,t}\phi_t(-\delta'(u_t))) = 0.$$

There are no changes in the production side. The government spends a fraction τ_t of the value-added output, and finances the subsidy by lump-sum tax. We assume that the government keeps the balanced-budget:

$$\underbrace{T_t}_{\text{tax revenue}} = \underbrace{\tau_t \mathcal{Y}_t}_{\text{government consumption}} + \underbrace{\tau_t^*(\sigma_e s_t)}_{\text{subsidy}}$$

Goods market clearing condition is

$$Y_t = \sigma_e c_t^e + \sigma_w c_t^w + \sigma_w i_t + \sigma_e s_t + N_t X_t + \tau_t \mathcal{Y}_t$$

Rearranging this equation, we obtain

$$\mathcal{Y}_t = \sigma_e c_t^e + \sigma_w c_t^w + \sigma_w i_t + \sigma_e s_t + \tau_t \mathcal{Y}_t \tag{61}$$

The value added output is the sum of consumption, investment in capital, investment in product developments, and government spending. Entrepreneur's budget constraint in the equilibrium is

$$c_t^e + p_{n,t}\left[(1-\theta)\vartheta_t s_t - \phi_t(1-\delta_n)N_t\right] - p_{k,t}\phi_t(1-\delta(u_t))K_t = \Pi_t N_t + R_t(u_t K_t) + (p_{n,t}\vartheta_t - (1-\tau_t^*))s_t - T_t. \tag{62}$$

The worker's budget constraint in the equilibrium is

$$
\begin{aligned}
& c_t^w + i_t + \frac{p_{n,t}}{\sigma_w}\left(N_{t+1} - \sigma_e\left[(1-\theta)\,\vartheta_t s_t + (1-\phi_t)(1-\delta_n)\,N_t\right]\right) \\
& + \frac{p_{k,t}}{\sigma_w}\left(1-\delta(u_t)\right)\left(1-\sigma_e(1-\phi_t)\right)K_t \\
= \; & \left[\Pi_t + p_{n,t}(1-\delta_n)\right]N_t + \left[R_t u_t + p_{k,t}\left(1-\delta(u_t)\right)\right]K_t + W_t l_t - T_t. \qquad (63)
\end{aligned}
$$

The model is summerized by the following equations:

$$Y_t = (u_t K_t)^{\alpha}(Z_t \sigma_w l_t)^{1-\alpha},$$

$$Z_t = \left(\bar{A}\right)(A_t N_t),$$

$$\psi_t l_t^{\zeta} = W_t\left(\frac{1}{c_t^w}\right),$$

$$W_t = (1-\xi)(1-\alpha)\frac{Y_t}{\sigma_w l_t},$$

$$\left(\frac{1}{c_t^e}\right) = (1+\lambda_t^*)\left(\frac{1}{c_t^w}\right),$$

$$\lambda_t^* = \frac{p_{n,t}\vartheta_t - (1-\tau_t^*)}{(1-\tau_t^*) - \theta p_{n,t}\vartheta_t},$$

$$p_{n,t} = E_t\left[\beta_t\left(\frac{c_t^w}{c_{t+1}^w}\right)\left(\Pi_{t+1} + p_{n,t+1}(1-\delta_n) + \sigma_e\lambda_{t+1}^*\left[\Pi_{t+1} + p_{n,t+1}(1-\delta_n)\,\phi_{t+1}\right]\right)\right],$$

$$\Pi_t = \left(\frac{\nu-1}{\nu}\right)\xi\frac{Y_t}{N_t},$$

$$R_t = (1-\xi)\,\alpha\frac{Y_t}{u_t K_t},$$

$$p_{k,t} = E_t\left[\beta_t\left(\frac{c_t^w}{c_{t+1}^w}\right)\left(R_{t+1}u_{t+1} + p_{k,t+1}\left(1-\delta(u_{t+1})\right) + \sigma_e\lambda_{t+1}^*\left[R_{t+1}u_{t+1} + p_{k,t+1}\left(1-\delta(u_{t+1})\right)\phi_{t+1}\right]\right)\right],$$

$$R_t + p_{k,t}\left(-\delta'(u_t)\right) + \sigma_e\lambda_t^*\left(R_t + p_{k,t}\phi_t\left(-\delta'(u_t)\right)\right) = 0,$$

$$1 = p_{k,t}\left(1 - \Lambda\left(\frac{i_t}{i_{t-1}}\right) - \Lambda'\left(\frac{i_t}{i_{t-1}}\right)\frac{i_t}{i_{t-1}}\right) + E_t\left[\beta_t\left(\frac{c_t^w}{c_{t+1}^w}\right)p_{k,t+1}\Lambda'\left(\frac{i_{t+1}}{i_t}\right)\left(\frac{i_{t+1}}{i_t}\right)^2\right],$$

$$
\begin{aligned}
& c_t^e + p_{n,t}\left[(1-\theta)\,\vartheta_t s_t - \phi_t(1-\delta_n)\,N_t\right] - p_{k,t}\phi_t\left(1-\delta(u_t)\right)K_t \\
= \; & \Pi_t N_t + R_t(u_t K_t) + \left(p_{n,t}\vartheta_t - (1-\tau_t^*)\right)s_t - \left[\tau_t\left(1-\frac{\xi}{\nu}\right)Y_t + \tau_t^*\sigma_e s_t\right],
\end{aligned}
$$

$$
\begin{aligned}
& c_t^w + i_t + \frac{p_{n,t}}{\sigma_w}\left(N_{t+1} - \sigma_e\left[(1-\theta)\vartheta_t s_t + (1-\phi_t)(1-\delta_n)N_t\right]\right) \\
& + \frac{p_{k,t}}{\sigma_w}(1-\delta(u_t))(1-\sigma_e(1-\phi_t))K_t \\
= \; & \left[\Pi_t + p_{n,t}(1-\delta_n)\right]N_t + \left[R_t u_t + p_{k,t}(1-\delta(u_t))\right]K_t + W_t l_t - \left[\tau_t\left(1-\frac{\xi}{\nu}\right)Y_t + \tau_t^* \sigma_e s_t\right],
\end{aligned}
$$

$$
\vartheta_t = \chi_t\left(\frac{\sigma_e s_t}{N_t}\right)^{\eta-1},
$$

$$
(1-\tau_t)\left(1-\frac{\xi}{\nu}\right)Y_t = \sigma_e c_t^e + \sigma_w c_t^w + \sigma_w i_t + \sigma_e s_t,
$$

$$
K_{t+1} = (1-\delta(u_t))K_t + \left(1-\Lambda\left(\frac{i_t}{i_{t-1}}\right)\right)(\sigma_w i_t).
$$

Detrended system is summarized as follows:

$$
\hat{Y}_t = \left(u_t\hat{K}_t\right)^\alpha \left(\left(\bar{A}\right)(A_t)\left[\sigma_w l_t\right]\right)^{1-\alpha},
$$

$$
\psi_t l_t^\zeta = \frac{\hat{W}_t}{\hat{c}_t^w},
$$

$$
\hat{W}_t = (1-\xi)(1-\alpha)\frac{\hat{Y}_t}{\sigma_w l_t},
$$

$$
\frac{1}{\hat{c}_t^e} = (1+\lambda_t^*)\frac{1}{\hat{c}_t^w},
$$

$$
\lambda_t^* = \frac{p_{n,t}\vartheta_t - (1-\tau_t^*)}{(1-\tau_t^*) - \theta p_{n,t}\vartheta_t},
$$

$$
p_{n,t} = E_t\left[\beta_t\left(\frac{1}{g_{t+1}}\frac{\hat{c}_t^w}{\hat{c}_{t+1}^w}\right)\left(\Pi_{t+1} + p_{n,t+1}(1-\delta_n) + \sigma_e\lambda_{t+1}^*\left[\Pi_{t+1} + p_{n,t+1}(1-\delta_n)\phi_{t+1}\right]\right)\right],
$$

$$
\Pi_t = \left(\frac{\nu-1}{\nu}\right)\xi\hat{Y}_t,
$$

$$
R_t = (1-\xi)\alpha\frac{\hat{Y}_t}{u_t\hat{K}_t},
$$

$$
p_{k,t} = E_t\left[\beta_t\left(\frac{1}{g_{t+1}}\frac{\hat{c}_t^w}{\hat{c}_{t+1}^w}\right)\left(R_{t+1}u_{t+1} + p_{k,t+1}(1-\delta(u_{t+1})) + \sigma_e\lambda_{t+1}^*\left[R_{t+1}u_{t+1} + p_{k,t+1}(1-\delta(u_{t+1}))\phi_{t+1}\right]\right)\right],
$$

$$
R_t + p_{k,t}(-\delta'(u_t)) + \sigma_e\lambda_t^*\left(R_t + p_{k,t}\phi_t(-\delta'(u_t))\right) = 0,
$$

$$
1 = p_{k,t}\left(1-\Lambda\left(g_t\frac{\hat{\imath}_t}{\hat{\imath}_{t-1}}\right) - \Lambda'\left(g_t\frac{\hat{\imath}_t}{\hat{\imath}_{t-1}}\right)\left(g_t\frac{\hat{\imath}_t}{\hat{\imath}_{t-1}}\right)\right) + E_t\left[\beta_t\left(\frac{1}{g_{t+1}}\frac{\hat{c}_t^w}{\hat{c}_{t+1}^w}\right)p_{k,t+1}\Lambda'\left(g_{t+1}\frac{\hat{\imath}_{t+1}}{\hat{\imath}_t}\right)\left(g_{t+1}\frac{\hat{\imath}_{t+1}}{\hat{\imath}_t}\right)^2\right],
$$

$$
\begin{aligned}
&\hat{c}_t^e + p_{n,t}\left[(1-\theta)\vartheta_t \hat{s}_t - \phi_t(1-\delta_n)\right] - p_{k,t}\phi_t(1-\delta(u_t))\hat{K}_t \\
= \;& \Pi_t + R_t\left(u_t \hat{K}_t\right) + \left(p_{n,t}\vartheta_t - (1-\tau_t^*)\right)\hat{s}_t - \left[\tau_t\left(1-\frac{\xi}{\nu}\right)\hat{Y}_t + \tau_t^*\sigma_e \hat{s}_t\right],
\end{aligned}
$$

$$
\begin{aligned}
&\hat{c}_t^w + \hat{\imath}_t + \frac{p_{n,t}}{\sigma_w}\left(g_{t+1} - \sigma_e\left[(1-\theta)\vartheta_t\hat{s}_t + (1-\phi_t)(1-\delta_n)\right]\right) \\
&+\frac{p_{k,t}}{\sigma_w}(1-\delta(u_t))(1-\sigma_e(1-\phi_t))\hat{K}_t \\
= \;& \Pi_t + p_{n,t}(1-\delta_n) + \left[R_t u_t + p_{k,t}(1-\delta(u_t))\right]\hat{K}_t + \hat{W}_t l_t - \left[\tau_t\left(1-\frac{\xi}{\nu}\right)\hat{Y}_t + \tau_t^*\sigma_e\hat{s}_t\right],
\end{aligned}
$$

$$
\vartheta_t = \chi_t(\sigma_e \hat{s}_t)^{\eta-1},
$$

$$
(1-\tau_t)\left(1-\frac{\xi}{\nu}\right)\hat{Y}_t = \sigma_e\hat{c}_t^e + \sigma_w\hat{c}_t^w + \sigma_w\hat{\imath}_t + \sigma_e\hat{s}_t,
$$

$$
g_{t+1}\hat{K}_{t+1} = (1-\delta(u_t))\hat{K}_t + \left(1-\Lambda\left(g_t\frac{\hat{\imath}_t}{\hat{\imath}_{t-1}}\right)\right)(\sigma_w\hat{\imath}_t).
$$

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