On the Convergence of Credit Risk in Current Consumer Automobile Loans

Jackson P. Lautier[1], Vladimir Pozdnyakov[2], Jun Yan[2]

[1]Department of Mathematical Sciences, Bentley University
[2]Department of Statistics, University of Connecticut

The Fifth Biennial Conference on Auto Lending
Federal Reserve Bank of Philadelphia
July 12, 2023

A junior credit analyst's observation

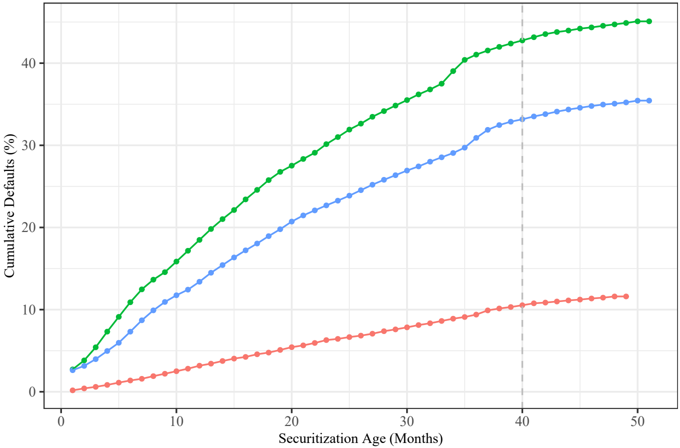


Figure: The growth rate of a cumulative ABS loss curve slows to an apparent equivalence between higher-risk or subprime pools (top curves) and lower-risk or prime pools (bottom curve).

BENTLEY UNIVERSITY

Table of Contents

Regulation AB

The Securities and Exchange Commission (SEC) recently implemented changes to the rules governing the issuance of asset-backed securities (ABS) (Securities and Exchange Commission, 2014, 2016).

Notably, it requires public issuers of ABS to make freely available pertinent loan-level information and payment performance on a monthly basis beginning in 2017.

The data in `xml` format may be accessed via the Electronic Data Gathering, Analysis, and Retrieval (EDGAR) system operated by the SEC.

Selected Bonds

We wrote `Python` code to scrape SEC filings to amass over 275,000 consumer automobile loans from the ABS bonds:

- CarMax Auto Owner Trust 2017-2 (CarMax, 2017);
- Ally Auto Receivables Trust 2017-3 (Ally, 2017);
- Santander Drive Auto Receivables Trust 2017-2 (Santander, 2017b);
- Drive Auto Receivables Trust 2017-1 (Santander, 2017a).

These four bonds were selected because:

(i) Taken together, they span the full consumer credit profile;

(ii) No issuer is a subsidiary of an auto manufacturer;

(iii) The paying periods span the same macroeconomic environment (i.e., actively paying starting in March-April-May 2017 for 44-52 months).

Loan selection

We filtered the original sample of 275,000+ loans to be as homogeneous as possible (aside from risk classification.)[1]

That is, no co-borrowers, same income underwriting level ("stated not verified), no subvention, used vehicles only, loan term (72-73 months), etc.

Number of loans left for statistical analysis: **58,118**.

[1] We are diversified to "noise" (i.e., largest geographic concentration: TX (13%); largest manufacturer concentration: Nissan (13%)), and we provide thorough sensitivity testing to loan selection parameters in the manuscript.

Risk-Based Pricing

Following Phillips (2013), a borrower's interest rate in risk band a, r_a, is

$$r_a = r_c + m + l_a,$$

where r_c is the cost of capital, m is the added profit margin, and l_a is a factor that varies by risk band. More generally,

$$l_a \equiv f(\text{PTI}, \%\text{Down}, \text{Loan AMT}, \text{Vehicle Val.}, \text{Credit}, \text{etc.}).$$

That is, the interest rate is the market's reflection of a borrower's (i.e., loan's) risk profile.

Risk band assignment

Hence, we can defer to the market and assign borrowers to risk bands via interest rate. Specifically,

Risk Band	APR Range	Count	Default%[2]
deep subprime	20%+	21,630	52%
subprime	15-20%	21,332	37%
near prime	10-15%	6,677	21%
prime	5-10%	6,300	10%
super prime	0-5%	2,179	4%
		58,118	

Note: The terms “deep subprime”, “subprime”, etc. also correspond well to the traditional credit score ranges (Consumer Financial Protection Bureau, 2019); see next slide.

[2]We define 3 consecutive months of missed payments = default.

Summary of 58,118 loans

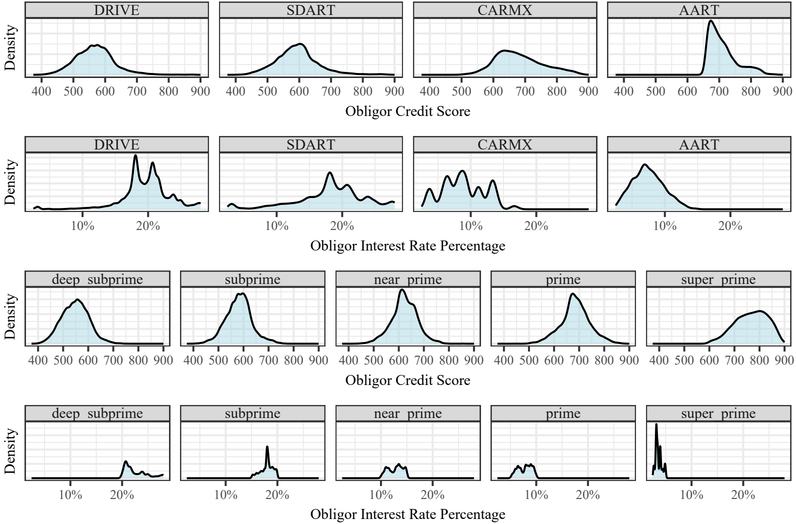


Figure: Details by bond, assigned risk band

Table of Contents

“Hazard”-ing a guess

Can we empirically demonstrate what appears to be an equivalence of credit risk, once a loan survives a minimal amount of time?

This is an ideal problem for the **hazard rate**,

$$\lambda(x) = \Pr(X = x \mid X \geq x),$$

because it is a *conditional* probability of default.

Goal (almost): Estimate $\lambda(x)$ by loan age and risk band for consumer loan data sampled from ABS.

Competing Risks

We desire to differentiate between a loan contract terminating due to default or due to prepayment.

Hence, our goal is to estimate the **cause-specific hazard rate**,

$$\lambda_i(x) = \Pr(X = x, Z_x = i \mid X \geq x),$$

from loans sampled from ABS, where Z_x denotes a random variable dependent on x in the spirit of a multistate process (e.g., Beyersmann et al., 2009).

Observe $\sum_i \lambda_i(x) = \lambda(x)$.

Statistical challenges of ABS data

We are not building a model. We are letting the data completely inform an estimate of the underlying cause-specific hazard rate distribution by risk band.

When estimating the time-to-event distribution from loans sampled from a securitization pool, however, there are incomplete data challenges due to *left-truncation*, *right-censoring*, and *discrete-time*.

These challenges for ABS data are rigorously studied in Lautier et al. (2023a) and Lautier et al. (2023b).

Estimating λ_{τ}^{0i} (competing risks with censoring)

Define $f_{*,\tau}^{0i}(x) = \Pr(X = x, X \leq C, Z_x = i \mid X \geq Y)$, $i = 1, 2$ and $U_\tau(x) = \Pr(Y \leq x \leq \min(X, C) \mid X \geq Y)$. It is may be shown

$$\lambda_{\tau}^{0i}(x) = \frac{\Pr(X = x, Z_x = i)}{\Pr(X \geq x)} = \frac{f_{*,\tau}^{0i}(x)}{U_\tau(x)}. \tag{1}$$

Estimation of (1) follows naturally with

$$\hat{f}_{*,\tau,n}^{0i}(x) = \frac{1}{n} \sum_{j=1}^{n} \mathbf{1}_{X_j \leq C_j} \mathbf{1}_{Z_{X_j} = i} \mathbf{1}_{\min(X_j, C_j) = x},$$

$$\hat{U}_{\tau,n}(x) = \frac{1}{n} \sum_{j=1}^{n} \mathbf{1}_{Y_j \leq x \leq \min(X_j, C_j)};$$

that is, $\hat{\lambda}_{\tau,n}^{0i}(x) = \hat{f}_{*,\tau,n}^{0i}(x) / \hat{U}_{\tau,n}(x)$, $i = 1, 2$.

Pleasingly, $\hat{\lambda}_{\tau,n}^{0i}(x)$ corresponds with classical treatments, such as Huang and Wang (1995), despite starting from the necessary ABS assumptions (discrete-time, sampling differences, etc.).

BENTLEY UNIVERSITY

Hypothesis test

Lautier et al. (2023c, Proposition 1, Lemma 1), which derive the asymptotic properties of $\hat{\lambda}^{0i}_{\tau,n}$, lead to a straightforward large sample financial econometric hypothesis test.

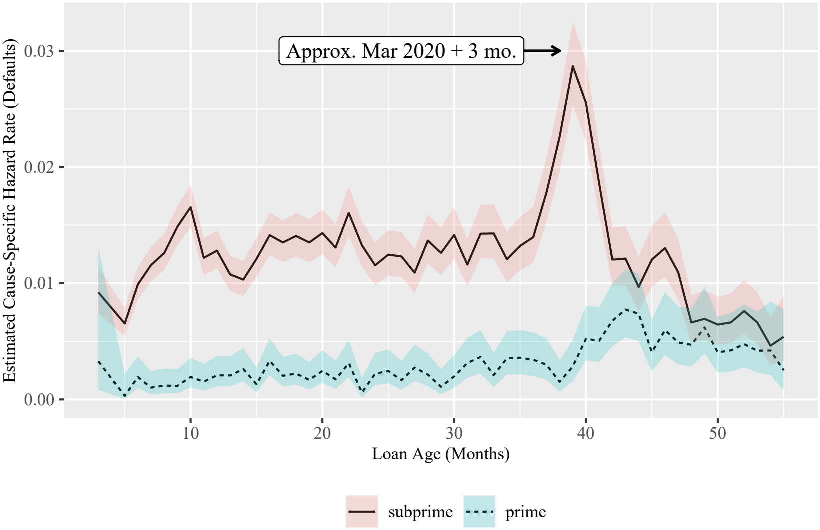
Specifically, let a, a' be two different risk bands (e.g., subprime vs. prime, etc.). Then we may test

$$H_0 : \lambda^{01}_{\tau,(a)} = \lambda^{01}_{\tau,(a')} \quad \text{vs.} \quad H_1 : \lambda^{01}_{\tau,(a)} \neq \lambda^{01}_{\tau,(a')},$$

for each age x by determining if the asymptotic confidence intervals in (2) overlap. Decision rule:

- Confidence intervals overlap $\implies$ fail to reject H_0 $\implies$ can't claim $\lambda^{01}_{\tau,(a)} \neq \lambda^{01}_{\tau,(a')}$ $\implies$ conditional default risk potentially converged.
- Confidence intervals **do not** overlap $\implies$ reject H_0 $\implies$ accept $\lambda^{01}_{\tau,(a)} \neq \lambda^{01}_{\tau,(a')}$ $\implies$ conditional default risk has **not** converged.

Credit Risk Convergence Visualization



Credit risk convergence matrix (months)

	deep sub.	subprime	near-prime	prime	sup.-prime
deep sub.	10	36	50	50	52
subprime		10	41	42	48
near-prime			10	13	43
prime				10	10
sup.-prime					10

Note: The first of three consecutive months of confidence interval overlap after month 10 for 72/73-month auto loans.

Robustness analysis

- COVID-19 plays some role but not the whole story (one-in-a-hundred-year events happen twice a decade...)
- Results hold for collateral type (new cars) and eliminating CarMax loans (different business model than banks)
- More analysis needed before generalizing to other loan types (auto loans generally considered to be a high *priority of payment* loan for consumers)

See Lautier et al. (2023c) for details.

Sometimes, a simple line plots suffices

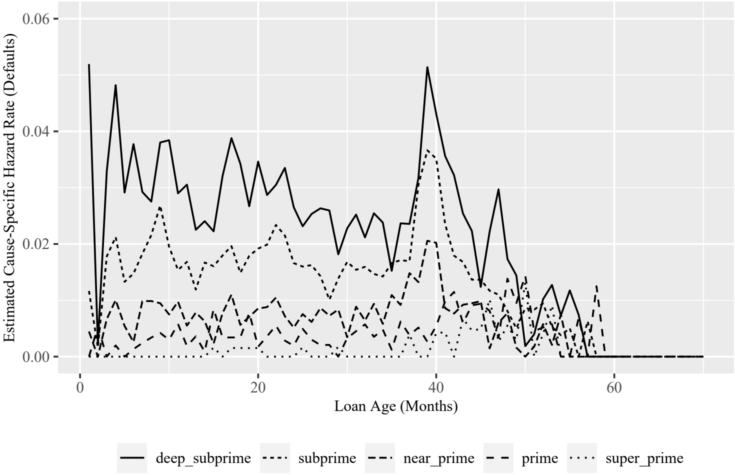


Figure: New cars, no CarMax, sans asymptotic confidence intervals

Table of Contents

BENTLEY UNIVERSITY

Lender Profits “Backloaded”

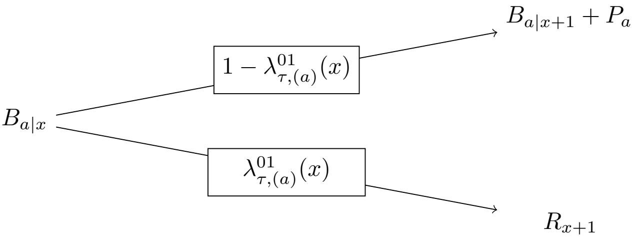
Backloaded profits necessary: Conventional wisdom is that the high-returns of high-risk loans that don’t default help repay the lender for the loans that do default.

The loans we consider are sampled from securitization pools, however, and so the risk has already been transferred off the lender’s books.[3]

Nonetheless, a risk-adjusted profitability analysis is instructive.

[3] ABS investors may be pricing in backloaded profits at the point of sale, but the main idea is that all our analysis is from a sample of loans the lender has already sold $\Longrightarrow$ lender portfolio level profitability not directly applicable here.

Deriving a month-by-month risk-adjusted return



Note: The assumed recovery, R_{x+1}, may be estimated from the ABS data.

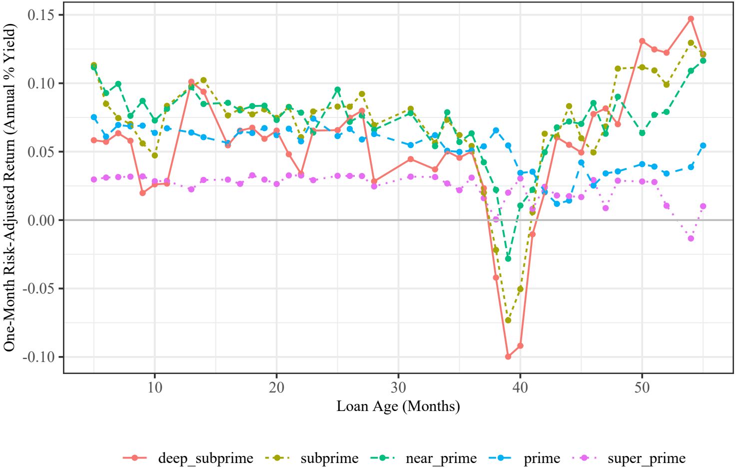


Figure: Est. conditional monthly risk-adjusted return

Estimated Savings (Deep Subprime Borrowers)

	Averages					Mo Pmt Savings ($)				Total Savings ($)			
Age	#	Bal	Pmt	APR(%)	# Pmts	S	NP	P	SP	S	NP	P	SP
12	17,558	14,245	365	22.58	65								
15	16,125	13,844	364	22.56	62								
18	14,375	13,520	363	22.54	60								
24	11,628	12,836	361	22.50	56								
30	9,492	11,973	361	22.46	50								
36	7,746	10,985	359	22.46	44	16				586			
42	6,050	9,833	357	22.46	38	16				490			
48	4,899	8,799	358	22.43	33	18				438			
50	4,622	8,312	358	22.44	30	12	33	52		267	729	1,153	
54	3,568	7,485	360	22.37	26	11	30	47	61	193	531	845	1,093
60	12	6,923	377	22.00	23	21	39	54	63	251	466	643	759

We find that deep subprime borrowers that remain current can maximize their savings by refinancing after about 48-50 months, when they converge in risk to prime/super prime borrowers.

Most current borrowers have prepaid by about loan age 60.

Estimated Savings (Subprime Borrowers)

		Averages				Mo Pmt Savings ($)				Total Savings ($)			
Age	#	Bal	Pmt	APR(%)	# Pmts	S	NP	P	SP	S	NP	P	SP
12	18,261	16,693	395	17.97	64								
15	17,021	16,126	394	17.96	61								
18	15,487	15,619	393	17.95	59								
24	12,997	14,621	389	17.94	54								
30	11,021	13,420	388	17.94	48								
36	9,309	12,194	386	17.94	42								
42	7,481	10,835	384	17.93	37		29	54			857	1,616	
48	6,192	9,506	383	17.92	31		22	44	61		526	1,055	1,473
50	5,901	8,953	383	17.93	29		23	44	60		508	963	1,325
54	4,542	7,975	386	17.94	25		22	40	55		389	723	988
60	22	7,021	414	17.47	20		25	40	50		299	477	596

We find that subprime borrowers that remain current can maximize their savings by refinancing after about 42 months, when they converge in risk to prime borrowers.

Again, most current borrowers have prepaid by about loan age 60.

Estimated Savings (Near-prime Borrowers)

		Averages				Mo Pmt Savings ($)				Total Savings ($)			
Age	#	Bal	Pmt	APR(%)	# Pmts	S	NP	P	SP	S	NP	P	SP
12	5,807	19,111	411	12.79	64								
15	5,587	18,245	407	12.76	60			39				2,206	
18	5,315	17,617	405	12.74	58			40				2,158	
24	4,692	16,204	402	12.72	52			35				1,657	
30	4,146	14,694	400	12.71	47			37				1,546	
36	3,592	13,187	398	12.71	41			31				1,116	
42	3,041	11,446	394	12.67	35			28				847	
48	2,622	9,862	394	12.68	29			21	39			494	928
50	2,455	9,283	395	12.69	27			20	37			436	811
54	1,663	8,218	400	12.69	24			29	44			526	798
60	63	6,435	413	11.98	17			13	22			160	269

We find that near-prime borrowers that remain current can maximize their savings by refinancing as soon as 15 months into the loan, when they converge in risk to prime borrowers.

Surprisingly, it appears many current near-prime borrowers follow a similar prepayment pattern as deep subprime, subprime borrowers (i.e., waiting until about loan age 60).

Estimated Savings (Prime Borrowers)

		Averages				Mo Pmt Savings ($)				Total Savings ($)			
Age	#	Bal	Pmt	APR(%)	# Pmts	S	NP	P	SP	S	NP	P	SP
12	5,173	18,582	358	7.83	64				39				2,327
15	5,283	17,611	354	7.81	60				33				1,880
18	5,315	16,706	350	7.78	57				30				1,627
24	4,971	15,097	346	7.76	52				32				1,535
30	4,538	13,503	345	7.74	46				30				1,245
36	4,096	11,866	344	7.73	39				21				755
42	3,697	10,274	342	7.72	34				23				703
48	3,191	8,615	343	7.71	28				21				513
50	2,963	8,101	345	7.71	26				21				460
54	1,898	7,075	351	7.66	22				18				324
60	92	4,756	328	7.38	16				22				261

We find that prime borrowers that remain current can maximize their savings by refinancing as soon as 12 months into the loan, when they converge in risk to super prime borrowers.

Surprisingly, it appears many current prime borrowers follow a similar prepayment pattern as deep subprime, subprime borrowers (i.e., waiting until about loan age 60).

BENTLEY UNIVERSITY

Analyzing Consumer Behavior

We can use the sibling cause-specific hazard rate estimator for prepayments, $\hat{\lambda}^{02}_{\tau,n}$, to analyze prepayment behavior by risk band.

We also overlay the Manheim Used Vehicle Value Index (ticker: `MUVVI`) and timing of the Economic Impact Payments for the 2017 and 2019 issuance.

Analyzing Consumer Behavior (Cont.)

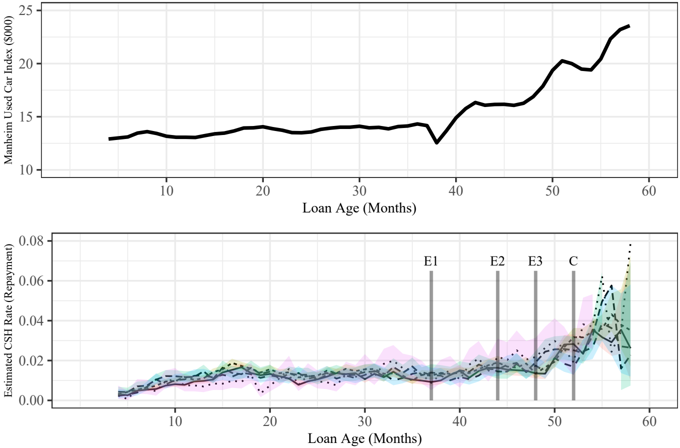


Figure: Conditional prepayment behavior by risk band

Table of Contents

Concluding thoughts

- *Consumerus Ignoramus?*: Consumers have a poor reputation in making financial decisions (e.g. Gross and Souleles, 2002; Stango and Zinman, 2011; Lusardi and de Bassa Scheresberg, 2013; Campbell, 2016; Heidhues and Kőszegi, 2016; Dobbie et al., 2021), but prepayments do accelerate as loans mature. Encourage borrowers to self-correct (questionable effectiveness (e.g., Keys et al., 2016; Agarwal et al., 2017)).
- *Financial innovation*: Lenders offer loans structured with a reducing payment based on good performance (may also act as an incentive to keep borrowers current keeping costs stable).
- *Competition*: Competing lenders seek out these mature loans to offer refinancing (similar to SOFI with student loans). That is, borrower delay possibly driven by perceived hassle, lack of options.
- *Regulation*: Require ongoing loans to be "re-underwritten" after a sustained period of good performance OR potentially offer borrowers cash rebates/larger trade-in values to refinance.

Thank you!

Acknowledgements We thank conference participants at the 2023 New England Statistics Symposium, the 2023 Boulder Summer Conference on Consumer Financial Decision Making at the University of Colorado Leeds School of Business, seminar participants at Bentley University, the University of Connecticut (School of Business, Department of Finance), the University of Connecticut (Department of Economics), Unversité Concordia University (John Molson School of Business, Department of Finance), and Joseph Golec, Brian Melzer, Jonathan A. Parker, and Jonathan Zinman. These acknowledgments should not be mistaken for endorsements; any remaining errors are the sole responsibility of the authors. Jackson P. Lautier was employed with Prudential Financial, Inc. from 2010-2019 but believes in good faith there are no conflicts of interest. The other authors have no potential conflicts to disclose. Jackson P. Lautier's work was supported by a National Science Foundation Graduate Research Fellowship under Grant No. DHE 1747453.

Jackson P. Lautier, FSA
e: jlautier@bentley.edu
w: www.jacksonlautier.com

BENTLEY UNIVERSITY

Table of Contents

BENTLEY UNIVERSITY

Asymptotic properties

(Lautier et al., 2023c, Proposition 1)

Define $\hat{\mathbf{\Lambda}}^{0i}_{\tau,n} = \left(\hat{\lambda}^{0i}_{\tau,n}(\Delta+1), \ldots, \hat{\lambda}^{0i}_{\tau,n}(\xi)\right)^\top$, where $\hat{\lambda}_{\tau,n}$, where $\hat{\lambda}^{0i}_{\tau,n}(x) = \hat{f}^{0i}_{*,\tau,n}(x)/\hat{U}_{\tau,n}(x)$, $i = 1, 2$. Then,

(i)

$$\hat{\mathbf{\Lambda}}^{0i}_{\tau,n} \xrightarrow{\mathcal{P}} \mathbf{\Lambda}^{0i}_{\tau}, \text{ as } n \to \infty;$$

(ii)

$$\sqrt{n}(\hat{\mathbf{\Lambda}}^{0i}_{\tau,n} - \mathbf{\Lambda}^{0i}_{\tau}) \xrightarrow{\mathcal{L}} N(0, \mathbf{\Sigma}^{0i}), \text{ as } n \to \infty,$$

where $\mathbf{\Lambda}^{0i}_{\tau} = \left(\lambda^{0i}_{\tau}(\Delta+1), \ldots, \lambda^{0i}_{\tau}(\xi)\right)^\top$ with $\lambda^{0i}_{\tau} = f^{0i}_{*,\tau}/U_\tau$ and

$$\mathbf{\Sigma} = \text{diag}\left(\frac{f^{0i}_{*,\tau}(\Delta+1)\{U_\tau(\Delta+1) - f^{0i}_{*,\tau}(\Delta+1)\}}{U_\tau(\Delta+1)^3}, \ldots, \frac{f^{0i}_{*,\tau}(\xi)\{U_\tau(\xi) - f^{0i}_{*,\tau}(\xi)\}}{U_\tau(\xi)^3}\right).$$

That is, the estimators $\hat{\lambda}^{0i}_{\tau,n}(\Delta+1), \ldots, \hat{\lambda}^{0i}_{\tau,n}(\xi)$ are consistent, asymptotically normal, and independent.

Asymptotic properties (cont.)

(Lautier et al., 2023c, Lemma 1)

The $(1-\theta)\%$ asymptotic confidence interval bounded within $(0,1)$ for $\lambda_\tau^{0i}(x)$, $x \in \{\Delta+1,\dots,\xi\}$, $i=1,2$ is

$$\exp\left\{\ln \hat{\lambda}_{\tau,n}^{0i}(x) \pm \mathcal{Z}_{(1-\theta/2)}\sqrt{\frac{\hat{U}_{\tau,n}(x)-\hat{f}_{*,\tau,n}^{0i}(x)}{n\hat{U}_{\tau,n}(x)\hat{f}_{*,\tau,n}^{0i}(x)}}\right\}, \tag{2}$$

where $\mathcal{Z}_{(1-\theta/2)}$ represents the $(1-\theta/2)$th percentile of the standard normal distribution.

Agarwal, Sumit, Itzhak Ben-David, and Vincent Yao, 2017, Systematic mistakes in the mortgage market and lack of financial sophistication, *Journal of Financial Economics* 123, 42–58.

Ally, 2017, Ally Auto Receivables Trust, Prospectus 2017-3, Ally Auto Assets LLC.

Beyersmann, Jan, Aurélien Latouche, Anika Buchholz, and Martin Schumacher, 2009, Simulating competing risks data in survival analysis, *Statistics in Medicine* 28, 956–971.

Campbell, John Y., 2016, Restoring rational choice: The challenge of consumer financial regulation, *American Economic Review* 106, 1–30.

CarMax, 2017, CarMax Auto Owner Trust, Prospectus 2017-2, CarMax Business Services LLC.

Consumer Financial Protection Bureau, 2019, Borrower risk profiles, url: https://www.consumerfinance.gov/data-research/consumer-credit-trends/auto-loans/borrower-risk-profiles/ (Accessed: 2022-06-15).

Dobbie, Will, Andres Liberman, Daniel Paravisini, and Vikram Pathania, 2021, Measuring bias in consumer lending, *The Review of Economic Studies* 88, 2799–2832.

Gross, David B., and Nicholas S. Souleles, 2002, Do liquidity constraints and interest rates matter for consumer behavior? Evidence from credit card data, *The Quarterly Journal of Economics* 117, 149–185.

Heidhues, Paul, and Botond Kőszegi, 2016, Naïeté-based discrimination, *The Quarterly Journal of Economics* 132, 1019–1054.

Huang, Ying, and Mei-Cheng Wang, 1995, Estimating the occurrence rate for prevalent survival data in competing risks models, *Journal of the American Statistical Association* 90, 1406–1415.

Keys, Benjamin J., Devin G. Pope, and Jaren C. Pope, 2016, Failure to refinance, *Journal of Financial Economics* 122, 482–499.

Lautier, Jackson, Vladimir Pozdnyakov, and Jun Yan, 2023a, Estimating a discrete distribution subject to random left-truncation with an application to structured finance, *Econometrics and Statistics* Accepted.

Lautier, Jackson, Vladimir Pozdnyakov, and Jun Yan, 2023b, Pricing time-to-event contingent cash flows: A discrete-time survival analysis approach, *Insurance: Mathematics and Economics* 110, 53–71.

Lautier, Jackson P., Vladimir Pozdnyakov, and Jun Yan, 2023c, On the convergence of credit risk in current consumer automobile loans, *arXiv* 1, 1–70.

Lusardi, Annamaria, and Carlo de Bassa Scheresberg, 2013, Financial literacy and high-cost borrowing in the United States, Working Paper 18969, National Bureau of Economic Research.

Phillips, Robert, 2013, Optimizing prices for consumer credit, *Journal of Revenue & Pricing Management* 12.

Santander, 2017a, Drive Auto Receivables Trust, Prospectus 2017-1, Santander Drive Auto Receivables LLC.

Santander, 2017b, Santander Drive Auto Receivables Trust, Prospectus 2017-2, Santander Drive Auto Receivables LLC.

Securities and Exchange Commission, 2014, 17 CFR Parts 229, 230, 232, 239, 240, 243, and 249 Asset-Backed Securities Disclosure and Registration.

Securities and Exchange Commission, 2016, 17 CFR §229.1125 (Item 1125) Schedule AL - Asset-level information.

Stango, Victor, and Jonathan Zinman, 2011, Fuzzy math, disclosure regulation, and market outcomes: Evidence from Truth-in-Lending reform, *The Review of Financial Studies* 24, 506–534.