Competition in the Financial Advisory Market:

Robo versus Traditional Advisors

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Prelimary version; Comments welcome

Abstract

We propose first a static and then a dynamic infinite-horizon model of the financial advisory market and show how to compute equilibrium solutions. Unregulated, market features are likely to result in high levels of concentration and a small number of dominant firms, with the speed of convergence towards dominance increased by the reinforcing interactions between firm size and profitability. Our goal is to provide regulators and market participants with a framework and tool kit that offers insights into how firm behavior in the financial advisory market may evolve as new digital technologies emerge and new entrants disrupt the market.

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1. Introduction

Over the past few years, technological innovation has transformed the financial advisory industry. With the advent of Robo Advisors, who provide digital platforms that connect investors with analytic tools to create financial plans or investment portfolios, low cost alternatives to traditional wealth managers have emerged. With over 40% of the global adult population still without access to financial services, digital advising is in a unique position to drive not only economic but also social change.[1]

Whether or not the emergence of Robo Advisors will lead to a permanent cost reduction for and broader access to financial advisory services will depend on the long-term level of competition in the asset management industry. The goal of our study is to develop a dynamic model of entry, competition and exit in the financial advisory market that provides a framework for discussing welfare implications and the impact of potential policy interventions.

Our framework captures many characteristics of the financial advisory market. First, we acknowledge that digital and human advising are not perfect substitutes. We therefore model the financial advisory market as a market for differentiated products, where the differentiation dimension is called "quality." There are two ways one could approach this: Fix the quality spectrum and have firms offer a price menu for different qualities, or have each firm choose a single price/quality combination. We will explore the latter. There is of course a trade-off as higher quality can fetch a higher price but is also costlier to produce.

Second, new entrants to the asset management industry face high start-up costs. We envision this entry cost to be the initial cost firms need to bear to match with clients, including advertising, collecting information, developing an initial algorithm and passing regulatory hurdles to become a financial advisor. Once firms enter into the industry, they continue to invest to match with clients, build a reputation, further their information stock and build up a client base. We call this stock value "client capital." Client capital can help firms reduce their cost of providing financial services, especially in quality. It also allows financial advisors to foster and improve client relationships. For example, firms can leverage client capital by reaching out to clients whose accounts are declining in value and they can reward desired client behavior such as a regular contribution of funds. We allow the technology of producing products to be heterogeneous across firms, so that firms that have the same client capital

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don't have to have the same cost of production.

Third, investors are heterogeneous in that they may have different preferences over "quality." Investors are willing to pay more for higher quality but there will be a threshold price where they will be willing to switch to another product. For example, Robo Advisors may lower the price so much that some clients switch to using their financial products, rather than going with traditional advisors. Investors also have the option of simply not purchasing anything.

Unregulated, these market features are likely to result in high levels of concentration and a single or small number of dominant firms, with the speed of convergence towards dominance increased by the reinforcing interactions between size and profitability of financial advisors: More clients generate more client data with can be mined or sold to increase service quality or reduce user fees which in turn attracts more clients.

New regulation may be aimed at increasing the size of the Robo Advisor market, the quality of Robo Advisor platforms, competition among wealth management companies, or the utility generated for clients. We will use our model to discuss the pros and cons of introducing such regulation. Our goal is to provide regulators and market participants with a framework and tool kit that enables them to quantify the impact new regulation may have.

The remainder of the paper is structured as follows. In Section 2 we review the related literature, and in Section 3 we give an overview of the financial advisory market and discuss the recent growth of digital advicing. In Section 4 we propose a static model of the financial advisory market and in Section 5 we use this framework to show that regulation that aims at lowering fixed costs for new market entrants has a positive impact on the accessibility of wealth management services. Section 6 describes the dynamic infinite-horizon extension of our model.

2. Related Literature

The academic finance and economics literature on Robo Advisors is still very thin, with only a few working papers in circulation (McDonald and Gao (2016), Kim, Maurer, and Mitchell (2016)).[2] But there exist a fair number of general overview articles in practitioner-focused journals such as Lopez,

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[2]Fein (2015) examines, from a legal perspective, whether Robo Advisors in fact provide personal investment advice, minimize costs, and are free from conflicts of interest. In another legal brief, Baker and Dellaert (2017) identify a number of questions that regulators need to be able to answer about Robo Advisors, and the capacities that regulators need to develop in order to answer those questions.

Babcic, and Ossa (2015), Schacht (2015), Weber (2016) and Falcon and Scherer (2017). And FinTech in general, and Robo Advisors in particular, have received wide coverage in the main stream media. For example, the "News" category on Google Search records over 44,000 entries for the term "Robo Advisor."

To the best of our knowledge, there are no theoretical models of Robo Advisors and their disruption to the financial advisory market in the literature. Our study offers the first such framework. In our model, the specification of investor utility allows for substitution between income and quality (Tirole (1988)), and our interpretation of the marginal rate of substitution is analogous to models where consumers differ by their incomes rather than by their tastes such as Gabszewicz and Thisse (1979 and 1980), Shaked and Sutton (1982, 1983, and 1984), Bonanno (1986), and Ireland (1987).

Computational methods for solving dynamic and repeated games have been developed in Judd, Yeltekin, and Conklin (2003), Sleet and Yeltekin (2016), and Yeltekin, Cai, and Judd (2017). Game-theoretic models of policy design can be found in Sleet and Yeltekin (2007), Sleet and Yeltekin (2008), Farhi, Sleet, Werning, and Yeltekin (2012), Judd, Schmedders, and Yeltekin (2012), and Bernheim, Ray, and Yeltekin (2015).

3. The Financial Advisory Market

A financial advisory company provides services to investors in exchange for a fee. As shown in Figure 1, the interaction between the financial advisor and its clients can be facilitated by a human advisor or a digital platform. The financial advisory company decides what service platform (or service quality) to offer and what service fee to charge its clients. Financial advisors also collect and analyze client data using in-house data miners, with the goal of improving the platform's quality and/or reducing costs. In addition, firms may buy additional data from third-party vendors or sell their client data to external data miners who want to analyze investor behavior.

Financial advisory services include guidance towards an appropriate asset allocation, selection of suitable investment products, development of tax efficient portfolio management strategies, estate planning services, and facilitation of the execution of client-directed trades. Financial advisors are expected to understand the client's investment needs and exert financial efforts to help investors meet their financial goals. The barriers to entering the financial advisory market include licensing, experience,
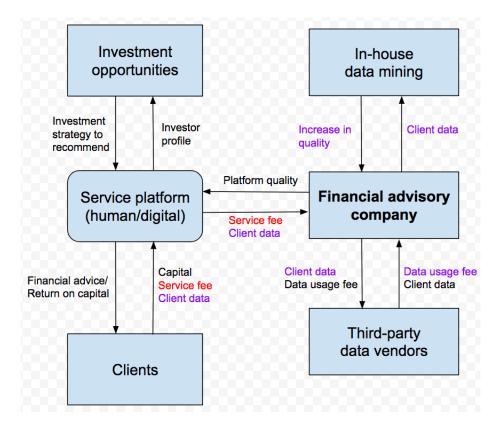
Figure 1: **Intermediation in the financial advisory market**

marketing and regulatory costs.

3.1 Traditional advisors

Traditional financial advisors offer services through a human professional who is trained and has experience with a variety of clients and life situations. These advisors tailor their clients' portfolio based on hard and soft information. In addition to investment advice, traditional financial advisors often also offer other services, such as estate planning, college savings and insurance advice. Traditional financial advisors generally charge fees in the range of 1% to 2% of clients' assets.

3.2 Robo advisors

A Robo Advisor is a company that provides an online platform through which it offers wealth management services. The platform provides financial advice based on a formula- and data-driven algorithm and operates with minimal human intervention. Robo Advisors emerged in 2008 with the

founding of Betterment LLC.[3]. Growth accelerated in the United States and other countries after 2011, and today there are over 100 RAs. Figure 2 shows the number of Robo Advisor launches in the United States between and 2015. Robo Advisors tend to charge lower fees than traditional advisors, at about 25 to 50 basis points.
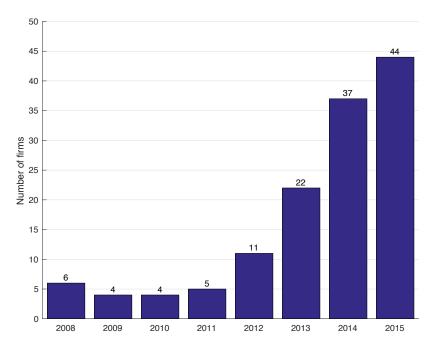

Figure 2: **Robo Advisor launches in the U.S.** The data were sourced from Tracxn Report: Robo Advisors (Feb. 2016) and BlackRock (2016).

KPMG estimated assets under management (AUM) for digital advice assets to be between $55-60 billion as of December 2015, which constituted only a very small portion of total U.S. retirement market assets of approximately $24 trillion.[4] The top U.S. Robo Advisors based on AUM as of December 2015 are shown in Figure 3. The top RAs include both independent start-ups and organizations that are part of larger firms providing asset management and/or brokerage services.

To better understand the evolution of the financial advisory market, we are interested in time series data on assets under management. Unfortunately, historical data are not available for all Robo Advisors. We argue, however, that we can proxy for AUM using the number of firms' Twitter followers. Figure 4 shows that there is a strong positive link between the number of Twitter followers of Robo

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[3]For a detailed history of Betterment, see www.betterment.com/resources/inside-betterment/our-story/the-history-of-betterment.
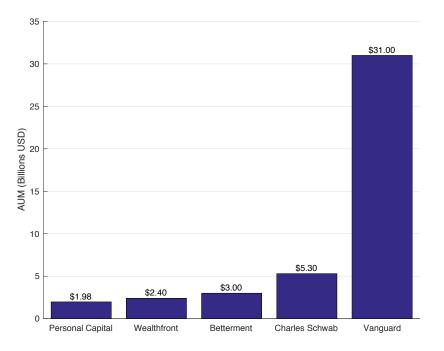[4]For details, see BlackRock (2016).

Figure 3: **Assets under management of top U.S. Robo Advisors** The figure shows the assets under management in billion USD as of December 2015 for top US Robo Advisors. The data were sourced from Tracxn Report: Robo Advisors (Feb. 2016) and BlackRock (2016).

Advisors and their AUM. Indeed, the R^2 of the log-linear fit is above 78%.

Using the number of Twitter followers as a proxy for AUM, Figure 5 visualizes the evolvement of Robo Advisors. Betterment enters first, but the in-house digital platforms of traditional wealth management services have since garnered the biggest market share. Nevertheless, a number of digital advisory firms have been competing over the last five years.

Robo Advisors often attract younger investors, such as millennials, who seek out low-cost services and are comfortable managing their portfolios online without personal advice. As these individuals mature and build assets, either through their own efforts and through their boomer parents and grandparents, they are likely to represent a significant growth opportunity. Indeed, over the next several decades, baby boomers are expected to pass down an estimated $30 trillion in assets to their children and grandchildren.[5]

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[5]For details, see www.cnbc.com/2016/11/29/preparing-for-the-30-trillion-great-wealth-transfer.html.
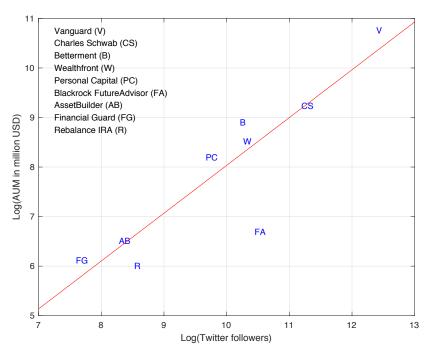
Figure 4: **Assets under management versus twitter followers of top US Robo Advisors** The figure shows the assets under management in million USD and number of twitter followers, logarithmic, for top US Robo Advisors. The data was sourced from Wikipedia and Twitter, and describes the market as of February 2017. For Vanguard and Charles Schwab, the assets under management are for their in-house Robo Advisors, i.e., for Vanguard Personal Advisor Services and Schwab Intelligent Portfolios.

4.   A Static Model of Competition in the Financial Advisory Market

Before we develop a fully dynamic model, we first analyze the static competition among financial advisors to highlight the basic trade-offs for market participants. Suppose advisors compete for clients in a differentiated product market by determining the quality and price pair. Clients choose among products based on their quality and price. As is typical in differentiated product market models, we assume that clients have the following indirect utility function:

$$u(\omega, p, \theta) = \theta \cdot \omega - p. \tag{1}$$

Here, ω represents the quality and p the price of the product. Clients differ in their taste, which is described by the parameter θ. We assume that θ has support on the interval $[0, \Theta]$, and use F_θ to denote its distribution.

The higher the quality ω of the good, the higher the utility reached by the client for any given price p. However, clients with a higher θ will be willing to pay more for a higher quality good. In accordance
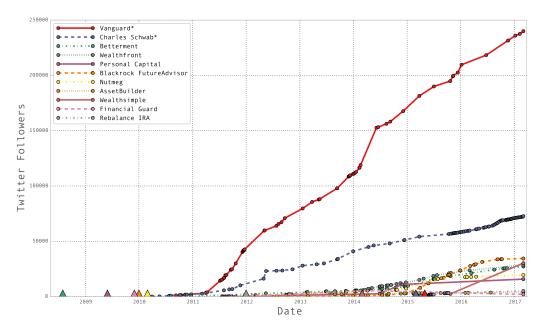
Figure 5: **Twitter followers of top U.S. Robo Advisors** The figure shows the time series of the number of Twitter followers of top U.S. Robo Advisors, from 2008 to 2017. Historical data are obtained from the Internet Archive website (https://archive.org/web/) which keeps caches of web pages from 2001 onwards. We use these caches to observe past Twitter accounts? websites and the number of Twitter followers on those days.

with the literature on product differentiation, we assume that clients can buy at most one unit of the good.[6] If clients decide not to buy the differentiated good, their utility is zero.

For this static exercise, we assume that there are only two financial advisory firms, and that they play the following two-stage game. In the first stage, they decide on the quality ω to be produced. We denote the two goods that will be produced by ω_1 and ω_2, and identify them by imposing $\omega_2 > \omega_1 \geq \underline{\omega}$.[7] If financial advisory firm i chooses to produce quality ω_1, we denote its product choice by $j(i) = 1$. If the firm instead chooses quality ω_2, then $j(i) = 2$. At this stage of the game, each firm incurs a fixed set-up cost c_i to produce quality $\omega_{j(i)}$.

In the second stage of the game, firms set prices. At this stage, the fixed set-up costs c_i have already been paid and product-dependent production costs c_j^p are incurred.

We look for the subgame perfect Nash equilibrium of the game. As usual, this will be obtained by

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[6]In the language of Tirole (1988), θ can be interpreted as the marginal rate of substitution between income and quality, so that a higher θ corresponds to a lower marginal utility of income and hence a higher income. Under this interpretation, the model proposed here is the analog of models where customers differ by their incomes rather than tastes (Gabszewicz and Thisse (1979 and 1980), Shaked and Sutton (1982, 1983, and 1984), Bonanno (1986), and Ireland (1987)).

[7]There is no a priori upper bound to the level of quality, but we assume that there exists a lower bound $\underline{\omega}$ to it. The latter can be interpreted as a minimum standard legal requirement or as being inherent to the production process.

backward induction. We first have to derive the equilibrium for the price-setting subgame, taking as fixed qualities ω_1 and ω_2. Clients that prefer buying good 2 over good 1 have a taste parameter θ such that $u(\omega_2, p_2, \theta) \geq u(\omega_1, p_1, \theta)$, or $\theta \geq (p_2 - p_1)/(\omega_2 - \omega_1)$. Clients who prefer buying good 1 over not buying at all have a taste parameter θ such that $0 \leq u(\omega_1, p_1, \theta) < u(\omega_2, p_2, \theta)$, or $(p_2 - p_1)/(\omega_2 - \omega_1) > \theta \geq p_1/\omega_1$. For clients $\theta = p_1/\omega_1$, the purchase of the good of quality ω_1 will imply zero utility. And clients with $\theta < p_1/\omega_1$ will not buy the differentiated product at all. In that sense, the financial advisory market is not covered.

The demand function can then be easily built. Given the price vector $\mathbf{p} \equiv (p_1, p_2)$ and quality pair $\omega \equiv (\omega_1, \omega_2)$ we can express the demands for different product types as follows:

$$
q_j(\mathbf{p}, \omega) = \begin{cases} 1 - F_\theta \left(\dfrac{p_2 - p_1}{\omega_2 - \omega_1} \right), & j = 2, \\[2mm] F_\theta \left(\dfrac{p_2 - p_1}{\omega_2 - \omega_1} \right) - F_\theta \left(\dfrac{p_1}{\omega_1} \right), & j = 1. \end{cases} \tag{2}
$$

Firms choose prices to maximize their stage-two profits $\pi_i = p_{j(i)} q_{j(i)} - c_{j(i)}^p$, for the given quality pair ω. The associated first-order conditions are:

$$
\frac{\partial \pi_i}{\partial p_{j(i)}} = 0, \quad i = 1, 2. \tag{3}
$$

From these conditions we can derive the equilibrium prices charged by the high and the low quality firm by solving the following two-equation system for $\mathbf{p}$ as a function of ω:

$$
1 - F_\theta \left(\frac{p_2 - p_1}{\omega_2 - \omega_1} \right) + p_2 F_\theta' \left(\frac{p_2 - p_1}{\omega_2 - \omega_1} \right) \frac{1}{\omega_2 - \omega_1} - \frac{\partial c_2^p}{\partial p_2} = 0, \tag{4}
$$

$$
F_\theta \left(\frac{p_2 - p_1}{\omega_2 - \omega_1} \right) - F_\theta \left(\frac{p_1}{\omega_1} \right) - p_1 F_\theta' \left(\frac{p_2 - p_1}{\omega_2 - \omega_1} \right) \frac{1}{\omega_2 - \omega_1} - p_1 F_\theta' \left(\frac{p_1}{\omega_1} \right) \frac{1}{\omega_1} - \frac{\partial c_1^p}{\partial p_1} = 0. \tag{5}
$$

In the special case where the distribution function $F_\theta(x)$ is affine in x—as is the case for the uniform distribution—and production costs scale linearly with quantity, the system (4)-(5) will have a unique solution $\mathbf{p} = \mathbf{p}(\omega)$.

Given the solution to the price competition part of the game, we look for solutions to the quality

game. Firms choose their quality specification to maximize their overall profits

$$\Pi_i = \pi_i - c_i, \quad i = 1, 2. \tag{6}$$

The first- and second-order conditions of the profit maximization problem are

$$\frac{\partial \Pi_i}{\partial \omega_{j(i)}} = \frac{\partial (q_{j(i)} p_{j(i)})}{\partial \omega_{j(i)}} - \frac{\partial c_i}{\partial \omega_{j(i)}} = 0 \tag{7}$$

$$\frac{\partial^2 \Pi_i}{\partial \omega_{j(i)}^2} = \frac{\partial^2 (q_{j(i)} p_{j(i)})}{\partial \omega_{j(i)}^2} - \frac{\partial^2 c_i}{\partial \omega_{j(i)}^2} < 0, \tag{8}$$

for $i = 1, 2$. Equations (7) and (8) ensure that the solution $\omega^* = (\omega_1^*, \omega_2^*)$ represents a local maximum.

Finding a local maximum, however, does not guarantee that we found a Nash equilibrium. Additional incentive compatibility conditions must be satisfied. In other words, we need to make sure no financial advisor has an incentive to deviate from its quality choice. Let us imagine that firm i_h is producing the high quality, $j(i_h) = 2$, and that firm i_l is producing the low quality, $j(i_l) = 1$. To be sure that the candidate solution $\omega^* = (\omega_1^*, \omega_2^*)$ to system (7)–(8) is indeed an equilibrium solution, we also have to check that firm i_l has no incentive to "leapfrog" the rival firm i_h and itself produce the higher quality. In other words, we have to prove that producing ω_1^* is the optimal reply by firm i_l "from below" to the choice by the rival firm i_h to produce ω_2^*. Likewise, we have to prove that firm i_h has no incentive to deviate and produce a quality lower than that produced by firm i_l, meaning we have to prove that producing ω_2^* is the optimal reply "from above" to ω_1^* along the whole spectrum of possible qualities.

Formally, the following conditions have to be satisfied:

$$\Pi_{i_l}(\omega^*) \geq \Pi_{i_l}(\omega_{i_l} = \omega, \omega_{i_h} = \omega_2^*) \text{ for } \omega \leq \omega_2^* \text{ and} \tag{9}$$

$$\Pi_{i_l}(\omega^*) \geq \Pi_{i_l}(\omega_{i_h} = \omega_2^*, \omega_{i_l} = \omega) \text{ for } \omega \geq \omega_2^*; \tag{10}$$

$$\Pi_{i_h}(\omega^*) \geq \Pi_{i_h}(\omega_{i_h} = \omega, \omega_{i_l} = \omega_1^*) \text{ for } \omega \leq \omega_1^* \text{ and} \tag{11}$$

$$\Pi_{i_h}(\omega^*) \geq \Pi_{i_h}(\omega_{i_l} = \omega_1^*, \omega_{i_h} = \omega) \text{ for } \omega \geq \omega_1^*. \tag{12}$$

Expressions (9)-(10) give the conditions for the optimal reply of the firm producing the low quality,

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while expressions (11)-(12) give the conditions for the optimal reply of the firm producing the high quality. Note that expression (10) ensures that the low-quality firm has no incentive to "leapfrog" its rival by producing the highest quality itself, whereas the expression (12) ensures that the high-quality firm does not want to offer a lower quality than its low-quality rival.

Finally, we can derive the positions of the indifferent clients. Clients with $\theta = (p_2 - p_1)/(\omega_2 - \omega_1)$ are indifferent between products 2 and 1, and clients with $\theta = p_1/\omega_1$ are indifferent between product 1 and not buying the differentiated product at all. Therefore, clients with $\theta < p_1/\omega_1$ are not covered by the market. Note that with the particular utility specification (1), θ can be interpreted as the marginal rate of substitution between income and quality, so that a higher θ corresponds to a lower marginal utility of income and therefore a higher income. In that sense, the low-θ individuals not served by the market would be low-income individuals. If a social welfare function puts positive weight on these individuals, then lowering prices for a given quality would be welfare-improving and this is where regulation would have an impact.[8]

5. Welfare Implications from the Static Model

In the appendix, we solve the static model for the special case where θ is uniformly distributed on $[0, \bar{\theta}]$, production costs are product specific and scale linearly with quantity, $c_j^p = c\omega_j q_j$, and set-up costs are firm-specific,

$$c_i = \alpha_i \frac{\omega_{j(i)}^2}{2}, \tag{13}$$

where $\alpha_1 \geq \alpha_2 = 1$.

We show that for $\alpha_1 > \alpha_2$, firm 1 will produce the low-quality good. As α_1 decreases towards $\alpha_2 = 1$, product quality increases and a larger fraction of clients are being covered by the financial advisory market. The results are summarized in Table 1. We observe that a lower start-up cost for firm 1, i.e., a lower α_1, is associated not only with higher product quality but also with a larger fraction of the clients being covered by the financial advisory market. In this sense, regulation that aims at lowering fixed costs for new market entrants would have a positive impact on the accessibility of wealth

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[8]Note that instead of having $\theta \in [0, \bar{\theta}]$, we could have $\theta \in [\underline{\theta}, \bar{\theta}]$. Prices and cutoff θ's would be in relationship to the lower as well as the upper bound, but would not change the basic results of the game.

management services.

Table 1: **Solutions to the static game with heterogeneous fixed costs** The table reports the outcome of the static game for $\bar{\theta} = 10$, $c = 1$ and different values of α_1. q_0 described the demand for the outside option. θ_{10} identifies clients that are indifferent between product 1 and not buying the differentiated product at all. θ_{21} identifies clients that are indifferent between products 2 and 1.

α_1	ω_1	ω_2	p_1	p_2	q_0	q_1	q_2	θ_{10}	θ_{21}	Π_1	Π_2
1	0.39	2.05	1.14	9.90	0.29	0.24	0.47	2.91	5.28	0.1002	1.6035
2	0.22	2.03	0.68	10.41	0.31	0.23	0.46	3.06	5.37	0.0565	1.8090
3	0.15	2.03	0.48	10.63	0.31	0.23	0.46	3.12	5.41	0.0393	1.8858
4	0.12	2.03	0.37	10.74	0.31	0.23	0.46	3.15	5.43	0.0301	1.9256
5	0.10	2.03	0.30	10.81	0.32	0.23	0.46	3.17	5.45	0.0244	1.9500
6	0.08	2.03	0.26	10.87	0.32	0.23	0.45	3.18	5.45	0.0205	1.9663
7	0.07	2.02	0.22	10.90	0.32	0.23	0.45	3.19	5.46	0.0177	1.9782
8	0.06	2.03	0.20	10.93	0.32	0.23	0.45	3.20	5.47	0.0155	1.9871
9	0.05	2.03	0.18	10.96	0.32	0.23	0.45	3.20	5.47	0.0138	1.9940
10	0.05	2.03	0.16	10.98	0.32	0.23	0.45	3.21	5.47	0.0125	1.9995

6. Modeling Entry, Competition and Exit in the Financial Advisory Market

Having identified the basic trade-offs that face firms in a differentiated market, we now turn to the dynamic problem. Firm behavior documented in the financial advisory market shows that firms enter at different times and have evolving market power over time. Our ultimate goal is to develop a model of this market that can be used for policy intervention exercises. To fully understand the implications of any policy intervention in this market, we need a dynamic, infinite-horizon model that can give insights into how firms behave in the market today and how that behavior will evolve in the future as policies change and new technologies and new entrants alter the nature of the market.

We therefore provide a dynamic strategic model of competition among financial advisors with entry, exit, differentiated financial products and investment in attracting clients and developing a client base to deliver quality services at a lower price. In the dynamic set-up, we have each period divided into two sub-periods, as in the static version. In the first sub-period, firms choose their quality. In the second sub-period, firms compete on prices and decide how much client capital to invest in.

Unlike the static case, we do not constrain the model to have duopoly competition only. Instead, N firms each supply one good $j = j(i)$ with quality $\omega_{j(i)}$, and price $p_{j(i)}$. In other words, firms specialize in one product by choosing a quality and price combination (ω_j, p_j). Let $j = 1, \ldots, m \leq N$, index the

distinct qualities on offer, where products are identified via $\omega_m > \omega_{m-1} > \ldots > \omega_1 \geq \underline{\omega}$. Product $j = 0$ is assumed to deliver zero utility as before.

Each client has multiple options—buying one of the products from the menu of offerings or not buying a product at all—and chooses the one that maximizes her utility. This yields $0 \leq \theta_1 \leq \theta_2 \leq \ldots \leq \theta_m \leq 1$ such that clients in $[0, \theta_1)$ do not buy a product, clients in $[\theta_1, \theta_2)$ buy product 1, ..., and clients in $[\theta_m, 1]$ buy product m. This assumes that when clients are indifferent, they choose buying over not buying and higher quality product over others. It is straightforward to show that the marginal client θ_j who is indifferent between buying products j and $(j-1)$ is given by $\theta_j = (p_j - p_{j-1})/(\omega_j - \omega_{j-1})$. Similarly, the marginal client θ_1 who is indifferent between buying product 1 and not buying a product at all is given by $\theta_1 = p_1/\omega_1$.

Thus, given the price vector $\mathbf{p} = (p_1, \ldots, p_m)$ and quality vector $\omega = (\omega_1, \ldots, \omega_m)$, we can express the total client demand for product j as

$$
q_j(\mathbf{p}, \omega) = \begin{cases} 1 - F_\theta \left(\dfrac{p_m - p_{m-1}}{\omega_m - \omega_{m-1}} \right), & j = m; \\[3mm] F_\theta \left(\dfrac{p_{j+1} - p_j}{\omega_{j+1} - \omega_j} \right) - F_\theta \left(\dfrac{p_j - p_{j-1}}{\omega_j - \omega_{j-1}} \right), & 1 < j < m; \\[3mm] F_\theta \left(\dfrac{p_2 - p_1}{\omega_2 - \omega_1} \right) - F_\theta \left(\dfrac{p_1}{\omega_1} \right), & j = 1. \end{cases}
\tag{14}
$$

6.1 Firms and the timing of their decisions

We assume that each financial advisor has the technological capability to offer any quality, albeit at a cost that increases with quality. Firms compete along the price/quality dimension and in equilibrium unit prices for the same quality will be the same. We assume that firm i offers product $\omega_{j(i)}$ at a unit price of $p_{j(i)}$ at a firm-specific but constant unit cost of c_i.

In addition to competing along the price/quality dimension, firms also make investment decisions regarding their client capital. We think of client capital as the stock of information a firm has about potential clients. Client data include hard and soft information about potential investors' financial circumstances, their risk tolerance and utility function. Financial advisors can either collect such data from their existing client base or purchase the data from third-party vendors. Firms can mine client data to learn about investors' preferences and produce a higher quality product at the same cost or,

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equivalently, the same quality product at a lower cost.

We refer to the cumulative investment in client data that firm i has undertaken in the past as client capital k_i. For each firm, its next-period client capital depends upon its current-period client capital, the quality of the good it produces this period and the amount of data collection it chooses to invest in. Using subscript t to identify the current period t, firm i's client capital evolution can be written as

$$k_{t+1,i} = \left(1 + \delta\left(\omega_{t,j(i)}\right)\right) k_{t,i} + a_{t,i}, \tag{15}$$

where $a_{t,i}$ is the amount of data purchased from third-party vendors in the current period. We assume that $k_{t,i} \in K_i$, where K_i is compact for each firm i. The function $\delta(\omega) \geq 0$ represents the growth rate of existing client capital after providing a product of quality ω. With this specification, even though firms do pay quality costs every period, the quality costs can decline over time with investment.

The timing of decisions within each period is as follows. Firms come into the period with k_i, the client capital they accumulated so far. In Stage 1, they make a quality decision and pay the set-up cost c_i, where

$$c_i = c_i(\omega_{j(i)}, k_i).$$

In Stage 2, firms compete on prices. At this point, they have already made their quality choices and paid the set-up costs c_i. Given the firm's quality and price choices, the quantities of product j produced by firm i, $q_{j,i}$, have to add up to the total demand for product j:

$$q_j(\mathbf{p}, \omega) = \sum_{i=1}^{N} q_{j,i}(\mathbf{p}, \omega), \quad \text{for all } j, \tag{16}$$

where $q_j(\mathbf{p}, \omega)$ is given by Equation (14). In Stage 2, firms also decide how much to invest into client capital, i.e., they also set a_i.

6.2 Costs, revenues, profits

In Stage 2, firms incur a unit production cost $c_i^p = c^p(\omega_{j(i)})$ that is a function of product quality. They can also alter their client capital by purchasing data from third-party vendors. The unit cost of

adding to their client capital is c^a. As a result, firm i's Stage 2 profits are given by

$$\pi_i = q_{j(i),i}\left(p_{j(i)}(\mathbf{k},\omega) - c^p(\omega_{j(i)})\right) - a_i\,c^a. \tag{17}$$

When firms make their Stage 2 price choices, they must take into consideration the quality choices $\omega = (\omega_1, \ldots, \omega_N)$ and existing client capital $\mathbf{k} = (k_1, \ldots, k_N)$ of their competitors.

Stage 1 set-up costs $c_i = c_i(\omega_{j(i)}, k_i)$ are firm-specific functions of product quality and client capital. We assume that c_i is increasing in $\omega_{j(i)}$ and decreasing in k_i. Firm i's Stage 1 profits are given by

$$\Pi_i = \pi_i - c_i. \tag{18}$$

Equilibrium quantities are determined by the market clearing condition (16).

6.3 Notation

Before we move onto the equilibrium definition, we introduce some notation. In each period, firms choose product quality in sub-period 1, and price and investment in sub-period 2. Firms make these choices simultaneously with other firms. Although actions are observable after decisions are made, the simultaneity of the moves implies that firms must anticipate and internalize their competitors' action.

We use $s_i = \{\omega_{j(i)}, p_{j(i)}, a_i\}$ to denote the action choices of firm i, and assume that s_i is drawn from a finite set S_i. Elements of the set $S = \times_{i=1}^N S_i$ represent all possible combinations of player actions and are called *action profiles*. A particular action profile will be denoted by $\underline{s}$, and s_{-i} refers to that action profile when player i is excluded.

The vector of client capital, $\underline{\mathbf{k}} = (k_1, \ldots, k_N)$, evolves deterministically over time according to

$$\begin{aligned}
\underline{\mathbf{k}}_{t+1} &= \mathcal{K}(\underline{s}_t, \underline{\mathbf{k}}_t) \\
&= (1 + \delta\,(\underline{\omega}_t))\,\underline{\mathbf{k}}_t + \underline{\mathbf{a}}_t,
\end{aligned}$$

where $\mathcal{K} : S \times K \to K$, $\underline{\omega} = (\omega_{j(1)}, \ldots, \omega_{j(N)})$ and $\underline{\mathbf{a}} = (a_1, \ldots, a_N)$. The action space for the dynamic game is S^∞.

Firm i's average discounted payoffs from a specific sequence of states and action profiles are

$$(1 - \beta) \sum_{t=0}^{\infty} \beta^t \, \Pi_i(\underline{\mathbf{s}}_t, \underline{\mathbf{k}}_t), \tag{19}$$

where $\beta \in (0, 1)$ is the common discount factor amongst financial advisors.[9]

6.4 Histories and strategies

At time $t = 0$, the history of the game is denoted by $\mathbf{h}_0$. It contains just the initial client capital states $\underline{\mathbf{k}}_0$. For time $t > 0$, the history $\mathbf{h}_t$ of the game is a sequence $\{\underline{\mathbf{k}}_0, \{\underline{\mathbf{s}}_\tau, \underline{\mathbf{k}}_{\tau+1}\}_{\tau=0}^{t-1}\}$, which includes the vector of client capitals, quality, price and investment choices of all the firms prior to t and the client capital firms enter into the period t with. Let $H_t = H_t(\underline{\mathbf{k}}_0)$ denote the set of feasible t-period histories from a given $\mathbf{k}_0$.

A pure strategy for player i is a sequence of functions $\{g_{t,i}\}_{t=0}^{\infty}$ that map histories to actions, $g_{t,i} : H_t \to S_{t,i}$. A *strategy profile* is a sequence of functions $\{g_t\}_{t=0}^{\infty}$, where g_t maps from H_t to S_t. We use $g|h_t$ to denote the continuation strategy profile for the remainder of the game that follows the history h_t.

Suppose that per-period profits for each financial advisor i, Π_i, are bounded by $[\underline{\Pi}_i, \overline{\Pi}_i]$. In each state $\underline{\mathbf{k}}$, the set of equilibrium profits is then contained within the compact set $\Omega = \times_{i=1}^{N} [\underline{\Pi}_i, \overline{\Pi}_i]$. We define $\mathcal{P}^*$ as the set of all correspondences that map from $K \equiv \times_{i=1}^{N} K_i$ to some closed subset of Ω,

$$\mathcal{P}^* = \{W : K \rightrightarrows \Omega\}.$$

6.5 Equilibrium and its characterization

The equilibrium concept we employ for our dynamic game is *subgame-perfect equilibrium*, or SPE. In a dynamic game, at any history, the "remaining game" is called a subgame and can be regarded as a game of its own. In dynamic games, Nash equilibrium is too permissive because it imposes no optimality conditions in these subgames, opening the door to violations of sequential rationality. Subgame-perfection strengthens Nash equilibrium by imposing the sequential rationality requirement

---

[9]Note that average discounted payoffs can be decomposed into a convex combination of current period payoffs (with weight $1 - \beta$) and the average discounted payoffs for the rest of the game.

that behavior be optimal in all circumstances (i.e., subgames), both those that arise in equilibrium (as required by Nash equilibrium) and those that arise out of equilibrium.

Definition 1. *A strategy profile g is a subgame-perfect equilibrium if for any history $\mathbf{h}_t \in H_t$ the continuation strategy $g|h_t$ is a Nash equilibrium of the continuation game.*

Now we can formally define the subgame-perfect equilibrium payoff correspondence of our dynamic game.

Definition 2. *Let V^* denote the correspondence that maps the current state into the set of average discounted payoffs that can be sustained in pure SPE.*

In our formulation of the dynamic game, each subgame-perfect equilibrium payoff vector $v \in V^*$ is supported by a profile of actions a consistent with Nash play in the current period and a vector of continuation payoffs w that are themselves payoffs in some subgame-perfect equilibrium. The key to finding V^* involves defining a recursive operator that maps future SPE payoffs into current SPE payoffs.

In analogy with the Bellman operator of dynamic programming, this observation motivates the introduction of the operator $B^* : \mathcal{P}^* \to \mathcal{P}^*$. Let $W \in \mathcal{P}^*$. We define $B^*(W)_k$ to be the set of possible payoffs consistent with a Nash equilibrium profile $\underline{\mathbf{s}}$ in state $\underline{\mathbf{k}}$ today and continuation payoffs drawn from the set $W_{g(\underline{\mathbf{s}},\underline{\mathbf{k}})}$.[10] That is,

$$B^*(W)_{\underline{\mathbf{k}}} = \left\{ v \left| \begin{array}{l} w_{g(\underline{\mathbf{s}},\underline{\mathbf{k}})} \in W_{g(\underline{\mathbf{s}},\underline{\mathbf{k}})}, \underline{\mathbf{s}} \in S, \text{ and for each } i = 1,\ldots,N, \\[2mm] v_i = (1-\beta)\,\Pi_i(\underline{\mathbf{s}},\underline{\mathbf{k}}) + \beta\,w_{i,g(\underline{\mathbf{s}},\underline{\mathbf{k}})} \\[2mm] v_i \geq \max\limits_{S_i} \min\limits_{\{W_{(g(s_{-i},s',\underline{\mathbf{k}}))}\}} (1-\beta)\,\Pi_i(s_{-i},s',\underline{\mathbf{k}}) + \beta\,w_{i,g(s_{-i},s',\underline{\mathbf{k}})} \end{array} \right. \right\}, \quad (20)$$

where the prime superscript is used to indicate next period.

Note that a value v is in $B^*(W)_{\underline{\mathbf{k}}}$ if there is a continuation payoff profile $w_{g(\underline{\mathbf{s}},\underline{\mathbf{k}})} \in W_{g(\underline{\mathbf{s}},\underline{\mathbf{k}})}$ such that $v = (1-\beta)\,\Pi(\underline{\mathbf{s}},\underline{\mathbf{k}}) + \beta\,w_{g(\underline{\mathbf{s}},\underline{\mathbf{k}})}$ is the value of playing $\underline{\mathbf{s}}$ today and, for each i, firm i will choose to play $s_i = \{\omega_{j(i)}, p_{j(i)}, a_i\}$ because it believes that to do otherwise would yield a worse continuation payoff in

---

[10]Note that $B^*(W)_x$ describes the x component set of the $B^*(W)$ correspondence, not $B^*(W_x)$.

next period's state,

$$v_i \quad \geq \quad \max_{S_i} \min_{\{W_{(g(s_{-i}, s', \underline{\mathbf{k}}))}\}} (1 - \beta) \, \Pi_i(s_{-i}, s', \underline{\mathbf{k}}) + \beta \, w_{i, g(s_{-i}, s', \underline{\mathbf{k}})}. \tag{21}$$

Inequality (21) will be referred to as the *incentive compatibility constraint*.[11]

6.6   Solving the Dynamic Game

Because the equilibrium value correspondence of the dynamic game does not admit a closed-form solution, we will use the numerical procedures of Sleet and Yeltekin (2016) and Yeltekin, Cai, and Judd (2017) to compute equilibria.

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[11]In constructing the equilibrium of the dynamic game, we use the one-stage deviation principle for infinite horizon games, which provides a useful characterization of SPE. This principle applies to games where overall payoffs are a discounted sum of uniformly bounded stage payoffs, as is the case in our setting.

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A. Solutions of the Static Game with Zero Production Costs

In this appendix, we solve the static model in Section 4 for the special case where θ is uniformly distributed on $[0, \bar{\theta}]$. Under these assumptions, the demand function (2) takes on the simple form

$$
q_j(\mathbf{p}, \omega) = \begin{cases} 1 - \dfrac{1}{\bar{\theta}} \dfrac{p_2 - p_1}{\omega_2 - \omega_1}, & j = 2; \\[3mm] \dfrac{1}{\bar{\theta}} \dfrac{p_2 - p_1}{\omega_2 - \omega_1} - \dfrac{1}{\bar{\theta}} \dfrac{p_1}{\omega_1}, & j = 1. \end{cases} \tag{A.1}
$$

The first-order conditions (3) can then be written as

$$
\begin{aligned}
\frac{\partial \pi_2}{\partial p_2} &= 1 + \frac{1}{\bar{\theta}} \frac{p_1 - 2p_2}{\omega_2 - \omega_1} = 0, \\
\frac{\partial \pi_1}{\partial p_1} &= \frac{1}{\bar{\theta}} \frac{p_2 - 2p_1}{\omega_2 - \omega_1} - \frac{1}{\bar{\theta}} \frac{2p_1}{\omega_1} = 0.
\end{aligned}
$$

The first equation can be rewritten as

$$
p_1 = 2p_2 - \bar{\theta}(\omega_2 - \omega_1) \tag{A.2}
$$

which, when substituted into the second equation, implies

$$
(-3p_2 + 2\bar{\theta}(\omega_2 - \omega_1))\omega_1 - 4p_2(\omega_2 - \omega_1) + 2\bar{\theta}(\omega_2 - \omega_1)^2 = 0. \tag{A.3}
$$

Solving Equation (A.3) for p_2 and then substituting p_2 into Equation (A.2) yields the equilibrium prices charged by the high- and low-quality firm:

$$
\begin{aligned}
p_2(\omega) &= 2\,\bar{\theta}\,\omega_2 \frac{\omega_2 - \omega_1}{4\omega_2 - \omega_1}, \tag{A.4} \\
p_1(\omega) &= \bar{\theta}\,\omega_1 \frac{\omega_2 - \omega_1}{4\omega_2 - \omega_1}. \tag{A.5}
\end{aligned}
$$

As a result, stage-two profits $\pi_i = p_{j(i)} \, q_{j(i)}$ are given as

$$
\pi_i(\omega) = \begin{cases} \bar{\theta}\, 4\, (\omega_2 - \omega_1) \left(\frac{\omega_2}{4\omega_2 - \omega_1} \right)^2, & j(i) = 2 \\[3mm] \bar{\theta}\, \omega_1 \omega_2 (\omega_2 - \omega_1) \left(\frac{1}{4\omega_2 - \omega_1} \right)^2, & j(i) = 1. \end{cases}
$$

Suppose firm i's cost function is given by $c_i = \omega_{j(i)}^2/2$. Then the first-order conditions of the overall profit maximization problem (7) are:

$$\bar{\theta}\, 4\left(4\,\omega_2^2 - 3\,\omega_1\omega_2 + 2\,\omega_1^2\right) - (4\omega_2 - \omega_1)^3 = 0, \tag{A.6}$$

$$\bar{\theta}\,\omega_2^2\,(4\omega_2 - 7\omega_1) - \omega_1\,(4\omega_2 - \omega_1)^3 = 0. \tag{A.7}$$

Equations (A.6) and (A.7) can be rewritten as

$$4\,\omega_2^3 - 23\,\omega_1\omega_2^2 + 12\,\omega_1^2\omega_2 - 8\,\omega_1^3 = 0 \tag{A.8}$$

$$64\,\omega_2^3 - (48\omega_1 + 16\bar{\theta})\omega_2^2 + (12\omega_1^2 + \bar{\theta}\, 12\,\omega_1)\omega_2 - \omega_1^3 - \bar{\theta}\, 8\,\omega_1^2 = 0. \tag{A.9}$$

We solve the 2-equation system (A.8)–(A.9) as a function of $\bar{\theta}$. It is straightforward to show that the second-order conditions for profit maximization are met, and that the local maximum identified does indeed satisfy the incentive compatibility conditions (9)-(12) and hence represents a Nash equilibrium. The results are shown in Figure A.1. For a given $\bar{\theta}$, the solid curve shows pairs (ω_1, ω_2) that satisfy Equation (A.8) and the dashed curve shows pairs (ω_1, ω_2) that satisfy Equation (A.9). A candidate solution ω^* is obtained when the two curves intersect. We observe that a more disperse client distribution F_θ results in an increase in product quality.

Table A.1 reports additional results for the static game, including qualities, prices, quantities, and profits, as a function of $\bar{\theta}$. For $\bar{\theta} = 1$, the only solution in real numbers is: $\omega^* = (0.05, 0.25)$. In this scenario, clients with $\theta = 0.48$ are indifferent between product 2 and 1, and clients with $\theta = 0.21$ are indifferent between product 1 and not buying the differentiated product at all. In this scenario, 21% of clients are not covered by the market, 26% of clients buy the low-quality product and 53% of clients buy the high-quality product. The firm that produces the low-quality good makes no measurable profit, and the firm that produces the high-quality good makes a small profit of 0.02.

As the client distribution becomes more disperse and we observe a wider range for clients' marginal rate of substitution between income and quality, product quality, prices and firm profits increase. Note, however, that the percentages of clients not covered by the market and of clients preferring the low- or high-quality product remain the same. Independent of $\bar{\theta}$, 21% of clients opt out of the financial advisory market.
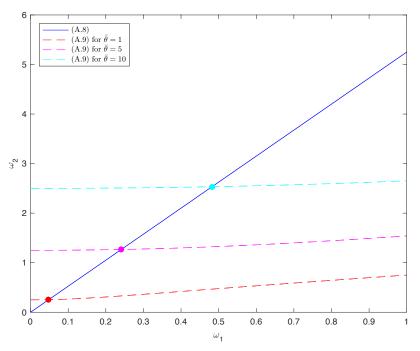
Figure A.1: **Quality choices in the static game** The figure shows solutions $\omega^* = (\omega_1^*, \omega_2^*)$ to the system (A.8)–(A.9), for different values of $\bar{\theta}$.

Table A.1: **Solutions to the static game** The table reports the outcome of the static game for different values of $\bar{\theta}$. q_0 described the demand for the outside option. θ_{10} identifies clients that are indifferent between product 1 and not buying the differentiated product at all. θ_{21} identifies clients that are indifferent between products 2 and 1.

$\bar{\theta}$	ω_1	ω_2	p_1	p_2	q_0	q_1	q_2	θ_{10}	θ_{21}	$\Pi_i, j(i) = 1$	$\Pi_i, j(i) = 2$
1	0.05	0.25	0.01	0.11	0.21	0.26	0.53	0.21	0.48	0.00	0.02
2	0.10	0.51	0.04	0.43	0.21	0.26	0.53	0.43	0.95	0.01	0.10
3	0.14	0.76	0.09	0.97	0.21	0.26	0.53	0.64	1.43	0.01	0.22
4	0.19	1.01	0.16	1.72	0.21	0.26	0.53	0.85	1.90	0.02	0.39
5	0.24	1.27	0.26	2.69	0.21	0.26	0.53	1.06	2.38	0.04	0.61
6	0.29	1.52	0.37	3.88	0.21	0.26	0.53	1.28	2.85	0.06	0.88
7	0.34	1.77	0.50	5.28	0.21	0.26	0.53	1.49	3.33	0.07	1.20
8	0.39	2.03	0.66	6.89	0.21	0.26	0.53	1.70	3.80	0.10	1.56
9	0.43	2.28	0.83	8.72	0.21	0.26	0.53	1.91	4.28	0.12	1.98
10	0.48	2.53	1.03	10.77	0.21	0.26	0.53	2.13	4.75	0.15	2.44
11	0.53	2.79	1.24	13.03	0.21	0.26	0.53	2.34	5.23	0.18	2.96
12	0.58	3.04	1.48	15.50	0.21	0.26	0.53	2.55	5.70	0.22	3.52
13	0.63	3.29	1.73	18.19	0.21	0.26	0.53	2.76	6.18	0.26	4.13
14	0.68	3.55	2.01	21.10	0.21	0.26	0.53	2.98	6.65	0.30	4.79
15	0.72	3.80	2.31	24.22	0.21	0.26	0.53	3.19	7.13	0.34	5.50
16	0.77	4.05	2.62	27.56	0.21	0.26	0.53	3.40	7.60	0.39	6.26
17	0.82	4.31	2.96	31.12	0.21	0.26	0.53	3.61	8.08	0.44	7.06
18	0.87	4.56	3.32	34.88	0.21	0.26	0.53	3.83	8.55	0.49	7.92
19	0.92	4.81	3.70	38.86	0.21	0.26	0.53	4.04	9.03	0.55	8.82
20	0.96	5.07	4.10	43.07	0.21	0.26	0.53	4.25	9.50	0.61	9.78

B. Solutions of the Static Game with Non-zero Production Costs

In this appendix, we solve the static model in Section 4 for the special case where θ is uniformly distributed on $[0, \bar{\theta}]$ and stage-two costs $c_j^p = c\,\omega_j\,q_j$. The demand function (2) remains the same as in (A.1). The first-order conditions (3) can then be written as

$$
\begin{aligned}
\frac{\partial \pi_2}{\partial p_2} &= 1 + \frac{1}{\bar{\theta}}\frac{p_1 - 2p_2}{\omega_2 - \omega_1} + c\frac{1}{\bar{\theta}}\frac{\omega_2}{\omega_2 - \omega_1} = 0, \\
\frac{\partial \pi_1}{\partial p_1} &= \frac{1}{\bar{\theta}}\frac{p_2 - 2p_1}{\omega_2 - \omega_1} - \frac{1}{\bar{\theta}}\frac{2p_1}{\omega_1} + c\frac{1}{\bar{\theta}}\frac{\omega_1}{\omega_2 - \omega_1} + c\frac{1}{\bar{\theta}} = 0.
\end{aligned}
$$

The first equation can be rewritten as

$$
p_1 = 2p_2 - \bar{\theta}(\omega_2 - \omega_1) - c\,\omega_2 \tag{B.1}
$$

which, when substituted into the second equation, implies

$$
(-3p_2 + 2\bar{\theta}(\omega_2 - \omega_1) + c(2\omega_2 + \omega_1))\omega_1 - 4p_2(\omega_2 - \omega_1) + 2\bar{\theta}(\omega_2 - \omega_1)^2 + c(2\omega_2 + \omega_1)(\omega_2 - \omega_1) = 0. \tag{B.2}
$$

Solving Equation (B.2) for p_2 and then substituting p_2 into Equation (B.1) yields the equilibrium prices charged by the high- and low-quality firm:

$$
\begin{aligned}
p_2(\omega) &= 2\,\bar{\theta}\,\omega_2\frac{\omega_2 - \omega_1}{4\omega_2 - \omega_1} + c\,(2\omega_2 + \omega_1)\,\omega_2\frac{1}{4\omega_2 - \omega_1}, \tag{B.3} \\
p_1(\omega) &= \bar{\theta}\,\omega_1\frac{\omega_2 - \omega_1}{4\omega_2 - \omega_1} + 3\,c\,\omega_1\,\omega_2\frac{1}{4\omega_2 - \omega_1}. \tag{B.4}
\end{aligned}
$$

As a result, stage-two profits $\pi_i = p_{j(i)}\,q_{j(i)} - c_{j(i)}^p$ are given as

$$
\pi_i(\omega) = \begin{cases} 4\,(\bar{\theta} - c)\,(1 - c/\bar{\theta})(\omega_2 - \omega_1)\left(\frac{\omega_2}{4\omega_2 - \omega_1}\right)^2, & j(i) = 2 \\ (\bar{\theta} - c)\,(1 - c/\bar{\theta})\,\omega_1\omega_2(\omega_2 - \omega_1)\left(\frac{1}{4\omega_2 - \omega_1}\right)^2, & j(i) = 1. \end{cases}
$$

Suppose firm i's fixed cost is given by $c_i = \omega_{j(i)}^2/2$. Then the first-order conditions of the overall

profit maximization problem (7) are:

$$\tilde{\theta}\, 4\, \left(4\, \omega_2^2 - 3\, \omega_1\omega_2 + 2\, \omega_1^2\right) - (4\omega_2 - \omega_1)^3 \;=\; 0, \tag{B.5}$$

$$\tilde{\theta}\, \omega_2^2\, (4\omega_2 - 7\omega_1) - \omega_1\, (4\omega_2 - \omega_1)^3 \;=\; 0, \tag{B.6}$$

where $\tilde{\theta} = (\bar{\theta} - c)\, (1 - c/\bar{\theta}) = (\bar{\theta} - c)^2/\bar{\theta}$. Equations (B.5) and (B.6) can be rewritten as

$$4\, \omega_2^3 - 23\, \omega_1\omega_2^2 + 12\, \omega_1^2\omega_2 - 8\, \omega_1^3 \;=\; 0 \tag{B.7}$$

$$64\, \omega_2^3 - (48\omega_1 + 16\tilde{\theta})\omega_2^2 + (12\omega_1^2 + \tilde{\theta}\, 12\, \omega_1)\omega_2 - \omega_1^3 - \tilde{\theta}\, 8\, \omega_1^2 \;=\; 0. \tag{B.8}$$

We solve the 2-equation system (B.7)–(B.8) as a function of $\bar{\theta}$ and c. It is straightforward to show that the second-order conditions for profit maximization are met, and that the local maximum identified does indeed satisfy the incentive compatibility conditions (9)-(12) and hence represents a Nash equilibrium. The results are shown in Figure B.1. For a given $\bar{\theta}$ and c, the solid curve shows pairs (ω_1, ω_2) that satisfy Equation (B.7) and the dashed curve shows pairs (ω_1, ω_2) that satisfy Equation (B.8). A solution ω^* is obtained when the two curves intersect. We observe that higher proportional production costs result in a decrease in chosen product quality.

Table B.1 reports additional results for the static game, including qualities, prices, quantities, and profits, as a function of c^p. For zero production costs ($c^p = 0$), 21% of clients are not covered by the market, 26% of clients buy the low-quality product and 53% of clients buy the high-quality product. As production costs increase, the costs of producing the same level of quality increase. As a result, firms choose to produce lower-quality goods. The relative quality of and demand for the high- versus low-quality good remain the same though.

If production costs increase with product quality, product 1 becomes more expensive relative to product 2.
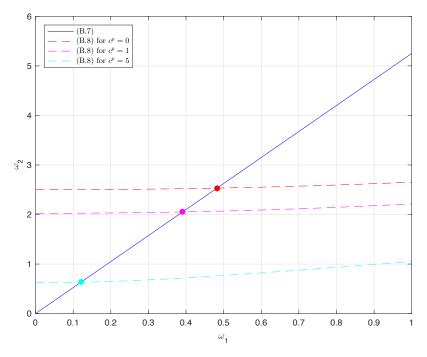
Figure B.1: **Quality choices in the static game with non-zero production costs** The figure shows solutions $\omega^* = (\omega_1^*, \omega_2^*)$ to the system (B.7)–(B.8), for $\bar{\theta} = 10$ and different values of c^p.

Table B.1: **Solutions to the static game with non-zero production costs** The table reports the outcome of the static game for $\bar{\theta} = 10$ and different values of c^p. q_0 described the demand for the outside option. θ_{10} identifies clients that are indifferent between product 1 and not buying the differentiated product at all. θ_{21} identifies clients that are indifferent between products 2 and 1.

c^p	ω_1	ω_2	p_1	p_2	q_0	q_1	q_2	θ_{10}	θ_{21}	$\Pi_i, j(i) = 1$	$\Pi_i, j(i) = 2$
0	0.48	2.53	1.03	10.77	0.21	0.26	0.53	2.13	4.75	0.1527	2.4438
1	0.39	2.05	1.14	9.90	0.29	0.24	0.47	2.91	5.28	0.1002	1.6035
2	0.31	1.62	1.14	8.75	0.37	0.21	0.42	3.70	5.80	0.0626	1.0010
3	0.24	1.24	1.06	7.42	0.45	0.18	0.37	4.49	6.33	0.0367	0.5867
4	0.17	0.91	0.92	5.97	0.53	0.16	0.32	5.28	6.85	0.0198	0.3167
5	0.12	0.63	0.73	4.51	0.61	0.13	0.26	6.06	7.38	0.0095	0.1527
6	0.08	0.41	0.53	3.12	0.69	0.11	0.21	6.85	7.90	0.0039	0.0626
7	0.04	0.23	0.33	1.89	0.76	0.08	0.16	7.64	8.43	0.0012	0.0198
8	0.02	0.10	0.16	0.90	0.84	0.05	0.11	8.43	8.95	0.0002	0.0039
9	0.00	0.03	0.04	0.24	0.92	0.03	0.05	9.21	9.48	0.0000	0.0000

C. Solutions of the Static Game with Heterogeneous Fixed Costs

In this appendix, we extend the analysis in the previous appendix to allow for a more flexible form of the fixed costs. Suppose firm 2's fixed cost is given by $c_2 = \omega_{j(i)}^2/2$, and firm 1's fixed cost is given by $c_1 = \alpha \omega_{j(i)}^2/2$, where $\alpha \geq 1$. In other words, firm 1 faces higher fixed costs than firm 2 to produce the same quality good. Then the first-order conditions of the overall profit maximization problem (7) depend on whether it is firm 1 that produces the low-quality good or not.

If firm 1 chooses ω_1, then

$$\tilde{\theta} \, 4 \left(4\,\omega_2^2 - 3\,\omega_1\omega_2 + 2\,\omega_1^2 \right) - (4\omega_2 - \omega_1)^3 = 0,$$

$$\tilde{\theta} \, \omega_2^2 \left(4\omega_2 - 7\omega_1 \right) - \alpha\,\omega_1 \left(4\omega_2 - \omega_1 \right)^3 = 0,$$

where $\tilde{\theta} = (\bar{\theta} - c)(1 - c/\bar{\theta}) = (\bar{\theta} - c)^2/\bar{\theta}$. These equations can be rewritten as

$$4\,\omega_2^3 - (7 + 16\alpha)\omega_1\,\omega_2^2 + 12\,\alpha\,\omega_1^2\omega_2 - 8\,\alpha\,\omega_1^3 = 0 \qquad (\text{C.1})$$

$$64\,\omega_2^3 - (48\omega_1 + 16\tilde{\theta})\omega_2^2 + (12\omega_1^2 + \tilde{\theta}\,12\,\omega_1)\omega_2 - \omega_1^3 - \tilde{\theta}\,8\,\omega_1^2 = 0. \qquad (\text{C.2})$$

If firm 1 chooses ω_2, then

$$\tilde{\theta} \, 4 \left(4\,\omega_2^2 - 3\,\omega_1\omega_2 + 2\,\omega_1^2 \right) - \alpha \left(4\omega_2 - \omega_1 \right)^3 = 0,$$

$$\tilde{\theta} \, \omega_2^2 \left(4\omega_2 - 7\omega_1 \right) - \omega_1 \left(4\omega_2 - \omega_1 \right)^3 = 0,$$

where $\tilde{\theta} = (\bar{\theta} - c)(1 - c/\bar{\theta}) = (\bar{\theta} - c)^2/\bar{\theta}$. These equations can be rewritten as

$$4\,\alpha\,\omega_2^3 - (7\alpha + 16)\omega_1\,\omega_2^2 + 12\,\omega_1^2\omega_2 - 8\,\omega_1^3 = 0 \qquad (\text{C.3})$$

$$64\,\alpha\,\omega_2^3 - (48\,\alpha\,\omega_1 + 16\tilde{\theta})\omega_2^2 + (12\,\alpha\,\omega_1^2 + \tilde{\theta}\,12\,\omega_1)\omega_2 - \alpha\,\omega_1^3 - \tilde{\theta}\,8\,\omega_1^2 = 0. \qquad (\text{C.4})$$

We solve the 2-equation systems (C.1)–(C.2) and (C.3)–(C.4) as a function of $\bar{\theta}, c$ and α. The results are shown in Figure C.1. For a given $\bar{\theta}$ and c, the solid curves shows pair (ω_1, ω_2) that satisfy Equation (C.1)/ (C.3) and the dashed curve shows pairs (ω_1, ω_2) that satisfy Equation (C.2)/ (C.4). A solution ω^* is obtained when the two curves intersect. We observe that lower fixed costs result in an
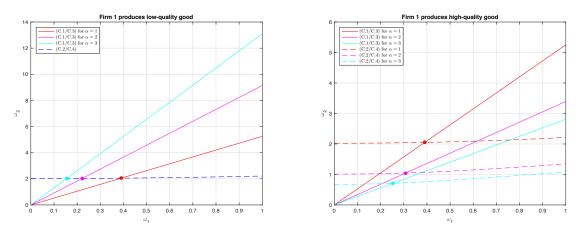
increase in chosen product quality.

Figure C.1: **Quality choices in the static game with heterogeneous fixed costs** The figure shows solutions $\omega^* = (\omega_1^*, \omega_2^*)$ to the system (C.1)–(C.2) (left plot) and system (C.3)–(C.4) (right plot), for $\bar{\theta} = 10$, $c = 1$ and different values of α.

Table C.1 reports additional results, including qualities, prices, quantities, and profits, as a function of α. We observe that overall profits are larger when firm 1 produces the low-quality product, meaning that this is the only practically relevant solution. (If firm 1 were to choose the high-quality product, firm 2 could always pay firm 1 to switch to the low-quality product and still be better off.)

Focusing on the scenario where firm 1 produces lower quality, we observe that a lower α is associated not only with higher product quality but also with a larger fraction of the clients being covered by the financial advisory market. In this sense, regulation that aims at lowering fixed costs for new market entrants would have an impact.

It is straightforward to show that the second-order conditions for profit maximization are met, and that the local maximum identified does indeed satisfy the incentive compatibility conditions (9)-(12) and hence represents a Nash equilibrium.

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Table C.1: **Solutions to the static game with heterogeneous fixed costs** The table reports the outcome of the static game for $\bar{\theta} = 10$, $c = 1$ and different values of α. q_0 described the demand for the outside option. θ_{10} identifies clients that are indifferent between product 1 and not buying the differentiated product at all. θ_{21} identifies clients that are indifferent between products 2 and 1.

α	ω_1	ω_2	p_1	p_2	q_0	q_1	q_2	θ_{10}	θ_{21}	Π_1	Π_2
Firm 1 produces low-quality good											
1	0.39	2.05	1.14	9.90	0.29	0.24	0.47	2.91	5.28	0.1002	1.6035
2	0.22	2.03	0.68	10.41	0.31	0.23	0.46	3.06	5.37	0.0565	1.8090
3	0.15	2.03	0.48	10.63	0.31	0.23	0.46	3.12	5.41	0.0393	1.8858
4	0.12	2.03	0.37	10.74	0.31	0.23	0.46	3.15	5.43	0.0301	1.9256
5	0.10	2.03	0.30	10.81	0.32	0.23	0.46	3.17	5.45	0.0244	1.9500
6	0.08	2.03	0.26	10.87	0.32	0.23	0.45	3.18	5.45	0.0205	1.9663
7	0.07	2.02	0.22	10.90	0.32	0.23	0.45	3.19	5.46	0.0177	1.9782
8	0.06	2.03	0.20	10.93	0.32	0.23	0.45	3.20	5.47	0.0155	1.9871
9	0.05	2.03	0.18	10.96	0.32	0.23	0.45	3.20	5.47	0.0138	1.9940
10	0.05	2.03	0.16	10.98	0.32	0.23	0.45	3.21	5.47	0.0125	1.9995
Firm 1 produces high-quality good											
1	0.39	2.05	1.14	9.90	0.29	0.24	0.47	2.91	5.28	1.6035	0.1002
2	0.31	1.05	0.84	4.64	0.27	0.24	0.49	2.71	5.14	0.6464	0.0808
3	0.25	0.71	0.66	2.97	0.26	0.25	0.49	2.59	5.06	0.3594	0.0674
4	0.21	0.54	0.54	2.17	0.25	0.25	0.50	2.51	5.01	0.2310	0.0577
5	0.18	0.44	0.45	1.71	0.25	0.25	0.50	2.45	4.97	0.1615	0.0504
6	0.16	0.37	0.39	1.40	0.24	0.25	0.51	2.41	4.94	0.1196	0.0448
7	0.15	0.32	0.35	1.19	0.24	0.25	0.51	2.38	4.92	0.0922	0.0403
8	0.13	0.28	0.31	1.03	0.24	0.25	0.51	2.35	4.90	0.0732	0.0366
9	0.12	0.25	0.28	0.91	0.23	0.26	0.51	2.33	4.89	0.0596	0.0335
10	0.11	0.23	0.25	0.82	0.23	0.26	0.51	2.31	4.88	0.0495	0.0309