FEDERAL RESERVE BANK of NEW YORK

Intermediary Leverage Cycles and Financial Stability

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Outline

Questions about Financial Stability Policy

- Systemic distress of financial intermediaries raises questions about financial stability policies:
    - How does capital regulation affect the trade-off between the pricing of credit and the amount of systemic risk?
    - How does macroprudential policy function in equilibrium?
    - What are the welfare implications of capital regulation?
- We develop a theoretical framework to address these questions

Our Approach

- We use a standard macro model with a financial sector and add one key assumption:
  - Funding constraints of financial intermediaries are risk based, so intermediaries have to hold more capital when the riskiness of their assets increases
- This assumption is empirically motivated from risk management practices and regulatory constraints

- Equilibrium dynamics capture stylized facts:
  - Procyclical leverage of intermediary balance sheets
  - Procyclical share of intermediated credit
  - Relationship between asset risk premia and intermediary leverage

Systemic Risk

Systemic risk return trade-off

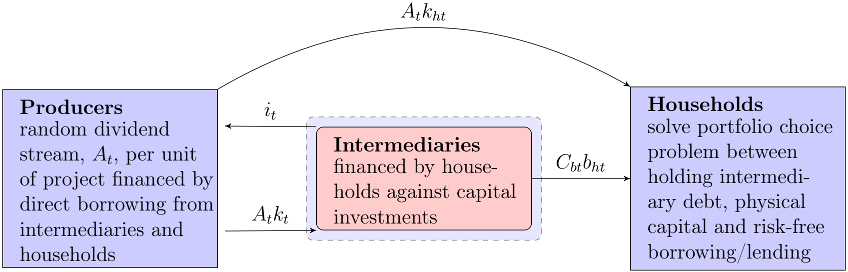
- Lower probability of distress corresponds to higher prices of risk
- Tightening capital requirements decreases probability of distress
- The relationship between household and capital requirements is inversely u-shaped

Volatility paradox

- Lower contemporaneous volatility is associated with higher probability of distress
- Lower volatility decreases effective risk aversion of intermediaries, leading to increased leverage and thus increased vulnerability to shocks

Outline

Economy Structure



Production

- Aggregate amount of capital K_t evolves as

$$dK_t = (I_t - \lambda_k)K_t dt$$

- Total output evolves as

$$Y_t = A_t K_t$$

- Stochastic productivity of capital $\{A_t = e^{a_t}\}_{t \geq 0}$

$$da_t = \bar{a}dt + \sigma_a dZ_{at}$$

- $p_{kt}A_t$ denotes the price of one unit of capital in terms of the consumption good

Households

- Household preferences are:

$$\mathbb{E}\left[\int_0^{+\infty} e^{-(\xi_t+\rho_h t)} \log c_t dt\right]$$

- Liquidity preference shocks (as in Allen and Gale (1994) and Diamond and Dybvig (1983)) are $\exp(-\xi_t)$

$$d\xi_t = \sigma_\xi \rho_{\xi,a} dZ_{at} + \sigma_\xi \sqrt{1-\rho_{\xi,a}^2} dZ_{\xi t}$$

- Households do not have access to the investment technology

$$dk_{ht} = -\lambda_k k_{ht} dt$$

Households' Optimization

$$\max_{\{c_t, k_{ht}, b_{ht}\}} \mathbb{E}\left[\int_0^{+\infty} e^{-(\xi_t+\rho_h t)} \log c_t dt\right]$$

subject to

$$dw_{ht} = r_{ft} w_{ht} dt + p_{kt} A_t k_{ht} \left(dR_{kt} - r_{ft} dt\right) + p_{bt} A_t b_{ht} \left(dR_{bt} - r_{ft} dt\right) - c_t dt$$

and no-shorting constraints

$$k_{ht} \geq 0$$
$$b_{ht} \geq 0$$

Intermediaries

- Financial intermediaries create new capital

$$dk_t = (\Phi(i_t) - \lambda_k)\, k_t dt$$

- Investment carries quadratic adjustment costs (Brunnermeier and Sannikov (2012))

$$\Phi(i_t) = \phi_0 \left(\sqrt{1 + \phi_1 i_t} - 1 \right)$$

- Intermediaries finance investment projects through inside equity and outside risky debt giving the budget constraint

$$p_{kt} A_t k_t = p_{bt} A_t b_t + w_t$$

Intermediaries' Risk Based Capital Constraint

- Risk based capital constraint (Danielsson, Shin, and Zigrand (2011))

$$\alpha \sqrt{\frac{1}{dt} \langle k_t d\left(p_{kt} A_t\right) \rangle^2} = w_t$$

- Implies a time-varying leverage constraint

$$\theta_t = \frac{p_{kt} A_t k_t}{w_t} = \frac{1}{\alpha \sqrt{\frac{1}{dt} \left\langle \frac{d(p_{kt} A_t)}{p_{kt} A_t} \right\rangle^2}}$$

- Note that the constraint is such that intermediary equity is proportional to the Value-at-Risk of total assets
- This will imply that default probabilities vary over time
- Microfoundation of the risk based capital constraint in a static setting is provided by Adrian and Shin (2010)

Risk-based Capital Constraints

VaR is the potential loss in value of inventory positions due to adverse market movements over a defined time horizon with a specified confidence level. We typically employ a one-day time horizon with a 95% confidence level.

Average Daily VaR

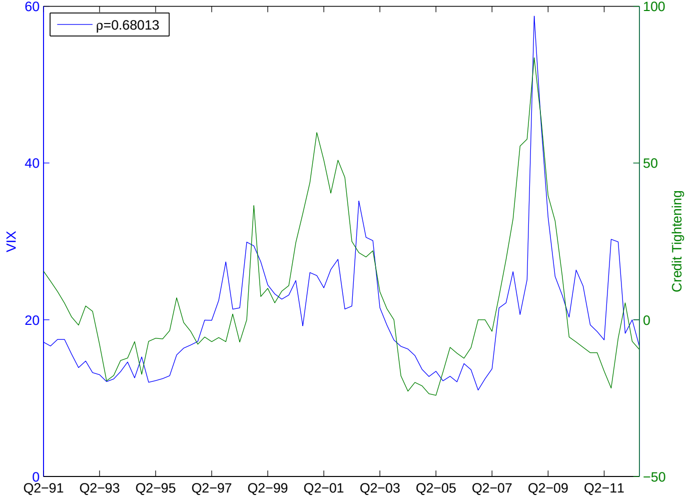
in millions	Year Ended December		
Risk Categories	**2011**	2010	2009
Interest rates	**$ 94**	$ 93	$176
Equity prices	**33**	68	66
Currency rates	**20**	32	36
Commodity prices	**32**	33	36
Diversification effect [1]	**(66)**	(92)	(96)
Total	**$113**	$134	$218

1. Equals the difference between total VaR and the sum of the VaRs for the four risk categories. This effect arises because the four market risk categories are not perfectly correlated.

Source: Goldman Sachs 2011 Annual Report

More

Commercial Bank Tightening Standards



Systemic Distress

- Solvency risk defined by

$$\tau_D = \inf_{t \geq 0} \{ w_t \leq \bar{\omega} p_{kt} A_t K_t \}$$

- Term structure of systemic distress

$$\delta_t(T) = \mathbb{P}(\tau_D \leq T | (w_t, \theta_t))$$

In distress

- Management changes
- Intermediary leverage reduced to $\underline{\theta} \approx 1$ by defaulting on debt
- Intermediary instantaneously restarts with wealth

$$w_{\tau_D^+} = \frac{\theta_{\tau_D}}{\underline{\theta}} w_{\tau_D}$$

Intermediaries' Optimization

- Intermediaries are myopic and maximize a mean-variance objective of instantaneous wealth

$$\max_{\theta_t, i_t} \quad \mathbb{E}_t\left[\frac{dw_t}{w_t}\right] - \frac{\gamma}{2}\mathbb{V}_t\left[\frac{dw_t}{w_t}\right],$$

subject to the dynamic intermediary budget constraint

$$dw_t = k_t p_{kt} A_t \left(dR_{kt} + \left(\Phi\left(i_t\right) - i_t/p_{kt}\right) dt\right) - b_t p_{bt} A_t dR_{bt}$$

and the risk based capital constraint

$$\alpha\sqrt{\frac{1}{dt}\left\langle k_t d\left(p_{kt} A_t\right)\right\rangle^2} = w_t$$

Equilibrium

An equilibrium in this economy is a set of price processes $\{p_{kt}, p_{bt}, C_{bt}\}_{t\geq 0}$, a set of household decisions $\{k_{ht}, b_{ht}, c_t\}_{t\geq 0}$, and a set of intermediary decisions $\{k_t, \beta_t, i_t, \theta_t\}_{t\geq 0}$ such that:

1. Taking the price processes and the intermediary decisions as given, the household's choices solve the household's optimization problem, subject to the household budget constraint.
2. Taking the price processes and the household decisions as given, the intermediary's choices solve the intermediary optimization problem, subject to the intermediary wealth evolution and the risk based capital constraint.
3. The capital market clears:
$$K_t = k_t + k_{ht}.$$
4. The risky bond market clears:
$$b_t = b_{ht}.$$
5. The risk-free debt market clears:
$$w_{ht} = p_{kt} A_t k_{ht} + p_{bt} A_t b_{ht}.$$
6. The goods market clears:
$$c_t = A_t \left(K_t - i_t k_t \right).$$

Related Literature

- **Leverage Cycles:** Geanakoplos (2003), Fostel and Geanakoplos (2008), Brunnermeier and Pedersen (2009)
- **Amplification in Macroeconomy:** Bernanke and Gertler (1989), Kiyotaki and Moore (1997)
- **Financial Intermediaries and the Macroeconomy:** Gertler and Kiyotaki (2012), Gertler, Kiyotaki, and Queralto (2011), He and Krishnamurthy (2012, 2013), Brunnermeier and Sannikov (2011, 2012)

Outline

Solution Strategy

- Equilibrium is characterized by two state variables, leverage θ_t and relative intermediary net worth ω_t

$$\omega_t = \frac{w_t}{w_t + w_{ht}} = \frac{w_t}{p_{kt} A_t K_t}$$

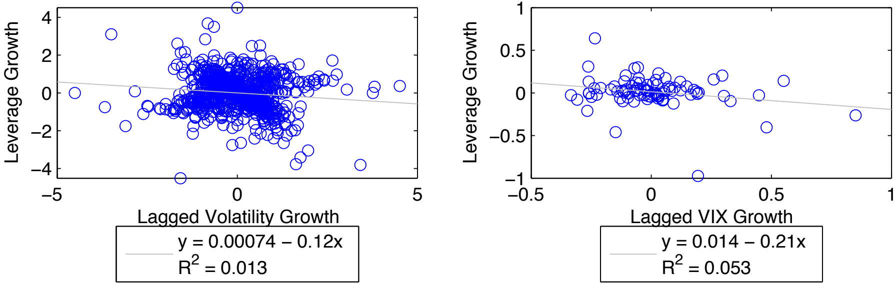
- Represent state dynamics as

$$\frac{d\omega_t}{\omega_t} = \mu_{\omega t} dt + \sigma_{\omega a,t} dZ_{at} + \sigma_{\omega \xi,t} dZ_{\xi t}$$
$$\frac{d\theta_t}{\theta_t} = \mu_{\theta t} dt + \sigma_{\theta a,t} dZ_{at} + \sigma_{\theta \xi,t} dZ_{\xi t}$$

- Risk-based capital constraint implies

$$\alpha^{-2} \theta_t^{-2} = \sigma_{ka,t}^2 + \sigma_{k\xi,t}^2$$

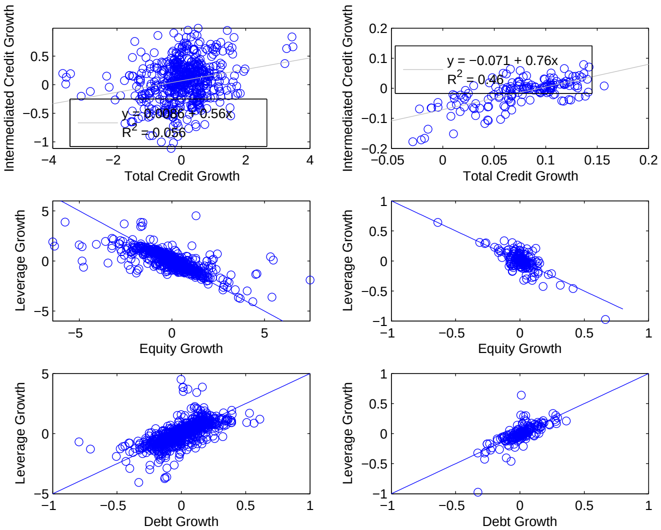
Volatility Risk



	Data	Mean	5%	Median	95%
β_0	0.014	0.000	-0.003	0.000	0.003
β_1	-0.208	-0.105	-0.187	-0.104	-0.025
R^2	0.053	0.013	0.001	0.011	0.035

Simulation details

Intermediary Balance Sheets I

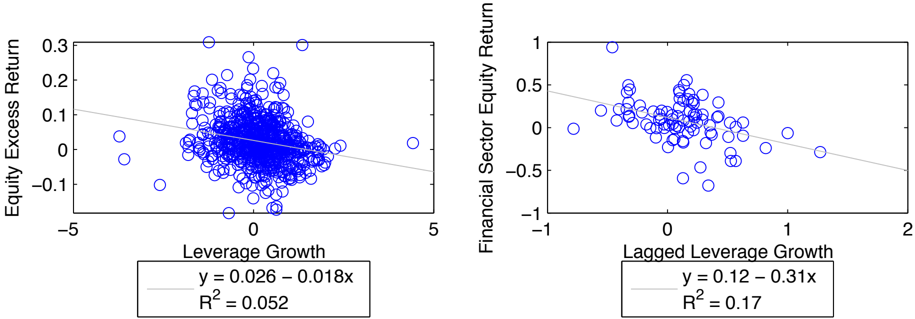


Intermediary Balance Sheets II

Table : Procyclicality of Intermediated Credit

	Data	Mean	5%	Median	95%
β_0	-0.071	-0.112	-0.203	-0.108	-0.040
β_1	0.756	0.434	0.190	0.433	0.680
R^2	0.460	0.048	0.009	0.045	0.101

Excess Returns



	Data	Mean	5%	Median	95%
β_0	0.118	0.076	0.068	0.076	0.084
β_1	-0.310	-0.031	-0.038	-0.031	-0.024
R^2	0.167	0.100	0.064	0.100	0.143

Equilibrium Prices of Risk I

Shocks

$$d\hat{y}_t = \sigma_a^{-1}\left(d\log Y_t - \mathbb{E}_t\left[d\log Y_t\right]\right) = dZ_{at}$$

$$\begin{aligned} d\hat{\theta}_t &= \left(\sigma^2_{\theta a,t} + \sigma^2_{\theta\xi,t}\right)^{-\frac{1}{2}}\left(\frac{d\theta_t}{\theta_t} - \mathbb{E}_t\left[\frac{d\theta_t}{\theta_t}\right]\right) \\ &= \frac{\sigma_{\theta a,t}}{\sqrt{\sigma^2_{\theta a,t} + \sigma^2_{\theta\xi,t}}} dZ_{at} + \frac{\sigma_{\theta\xi,t}}{\sqrt{\sigma^2_{\theta a,t} + \sigma^2_{\theta\xi,t}}} dZ_{\xi t}. \end{aligned}$$

Equilibrium Prices of Risk II

Price of risk of leverage

$$\eta_{\theta t} = \sqrt{1 + \frac{(\sigma_{ka,t} - \sigma_a)^2}{\sigma_{k\xi,t}^2}} \left(-\frac{2\theta_t \omega_t p_{kt}}{\beta (1 - \omega_t)} \sigma_{k\xi,t} + \sigma_\xi \sqrt{1 - \rho_{\xi,a}^2} \right)$$

- Price of risk of leverage is always positive (Adrian, Etula, and Muir (2013)), and depends on leverage growth in a nonmonotonic fashion (Adrian, Moench, and Shin (2010) find a negative relationship)

Equilibrium Prices of Risk III

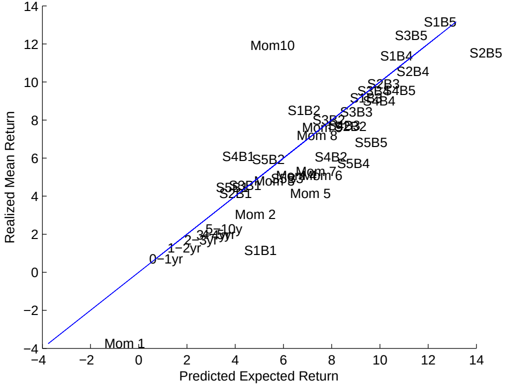


Figure : Source: Adrian, Etula, and Muir (2013)

Equilibrium Prices of Risk IV

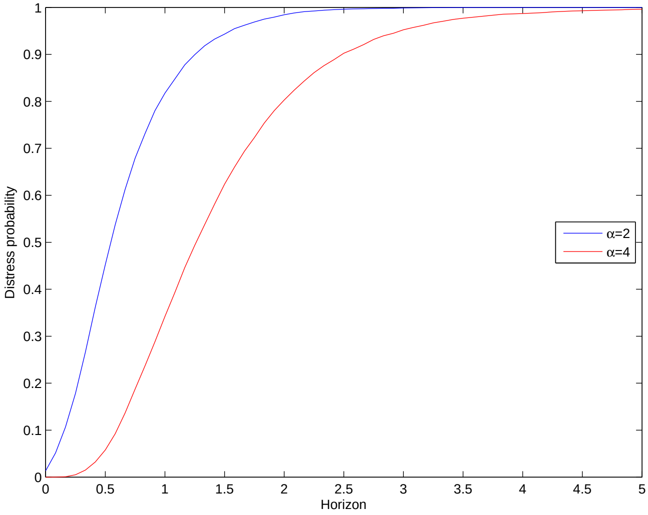
Price of risk of output

$$\eta_{yt} = \sigma_a + \sigma_\xi \left(\rho_{\xi,a} - \frac{\sigma_{ka,t} - \sigma_a}{\sigma_{k\xi,t}} \sqrt{1 - \rho_{\xi,a}^2} \right)$$

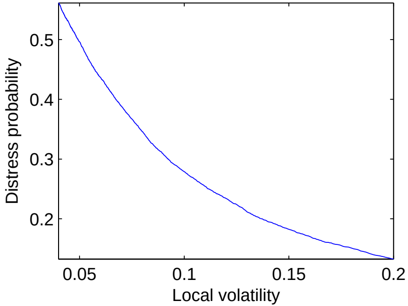
- Switches sign, consistent with insignificant estimates of price of output risk
- Usually becomes negative when exposure to liquidity shock is small

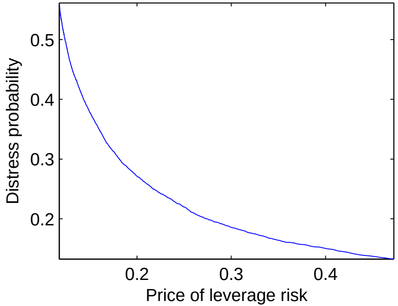
Outline

Term Structure of Systemic Risk



Volatility Paradox

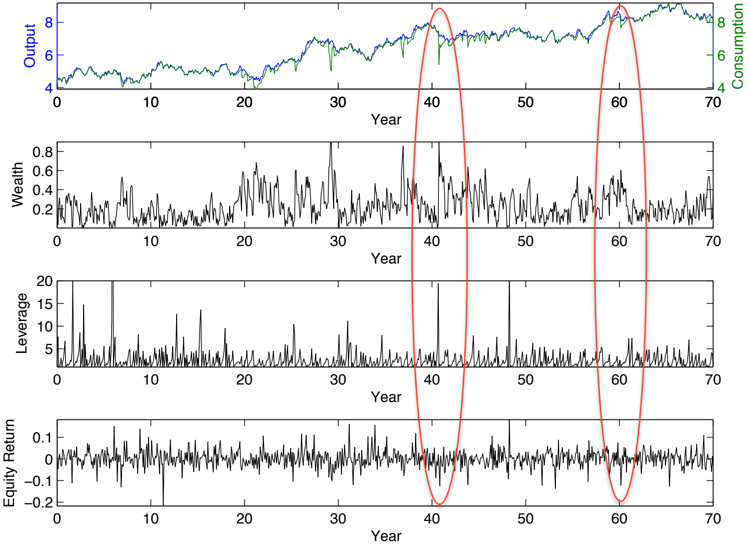




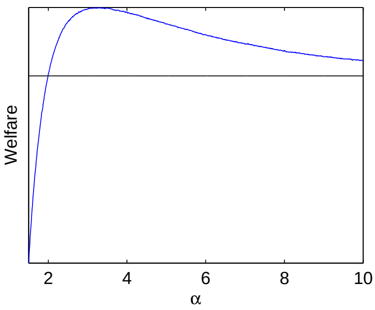
Constant Leverage Benchmark

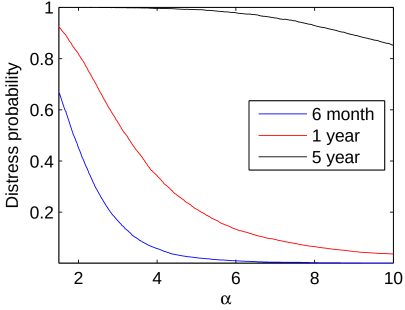
- Constant expected output and consumption growth
- But lower level of output and consumption growth
- Constant investment and price of capital
- Liquidity shocks have no impact on real activity

A Sample Path of the Economy



Household Welfare

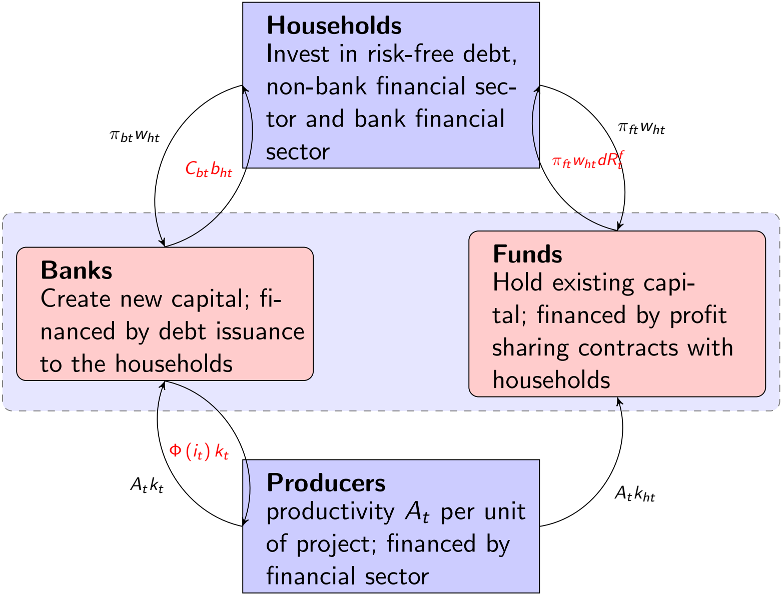




Outline

Alternative Specification

- Two financial sectors: banks and funds
- Leveraged intermediaries have VaR constraint (as in the current paper) while funds have skin in the game constraint (as in He and Krishnamurthy (2012, 2013))
- Bank managers, fund managers, and households have log utility
- VaR constraint sometimes binds


Households
Invest in risk-free debt, non-bank financial sector and bank financial sector
$\pi_{bt} w_{ht}$
$C_{bt} b_{ht}$
$\pi_{ft} w_{ht}$
$\pi_{ft} w_{ht} dR_t^f$
Banks
Create new capital; financed by debt issuance to the households
Funds
Hold existing capital; financed by profit sharing contracts with households
$\Phi(i_t) k_t$
$A_t k_t$
$A_t k_{ht}$
Producers
productivity A_t per unit of project; financed by financial sector

More

Additional Research

- Tradeoff between capital and liquidity regulation
- Stress tests
- Intermediation chains

▸ More

Conclusion

- Dynamic, general equilibrium theory of financial intermediaries' leverage cycle with endogenous amplification and endogenous systemic risk
- Conceptual basis for policies towards financial stability
- Systemic risk return trade-off: tighter intermediary capital requirements tend to shift the term structure of systemic downward, at the cost of increased prices of risk today
- Model captures important stylized facts:
  1. Procyclical intermediary leverage
  2. Procyclicality of intermediated credit
  3. Financial sector equity return and intermediary leverage growth
  4. Exposure to intermediary leverage shocks as pricing factor

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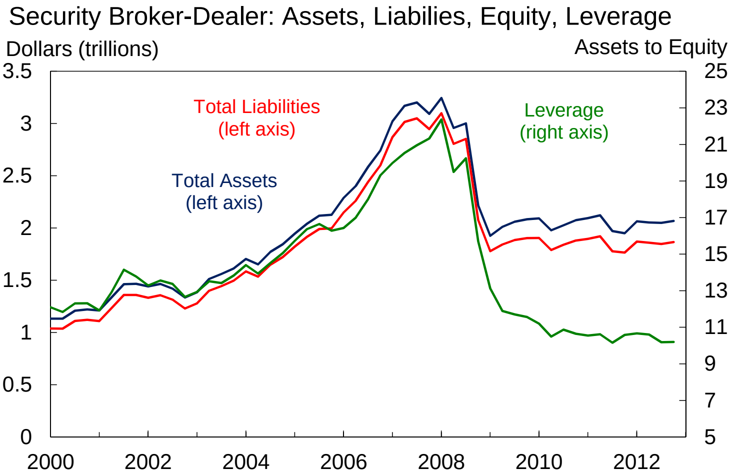
Outline

Broker-Dealer Balance Sheets: Levels



Source: Flow of Funds

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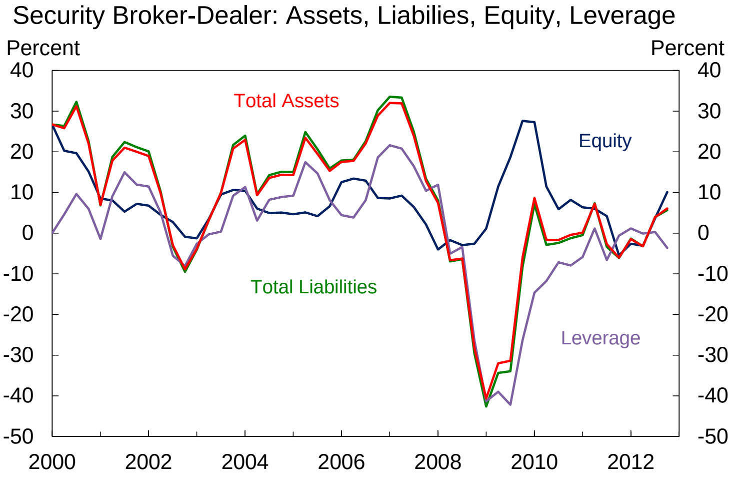
Broker-Dealer Balance Sheets: Annual Growth



Source: Flow of Funds

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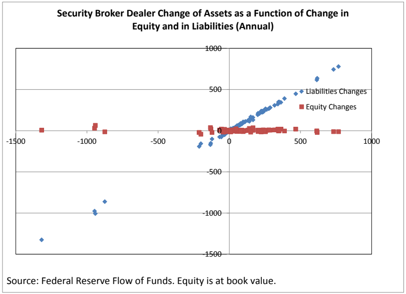
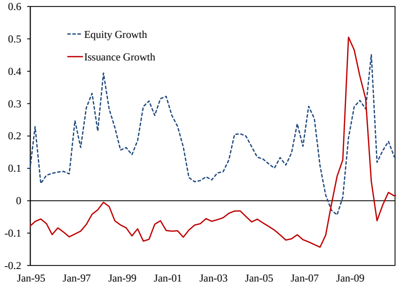
Broker-Dealer Balance Sheets: Adjustments





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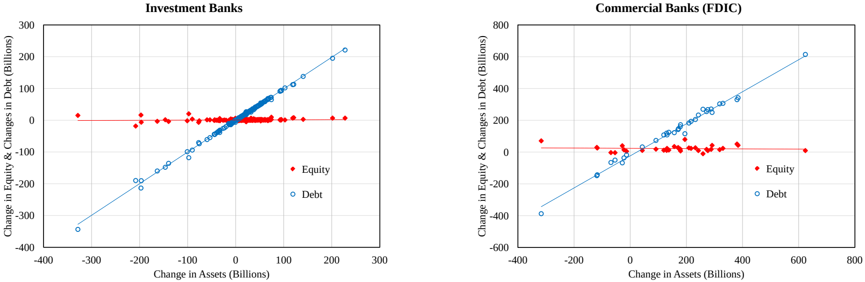
Balance Sheet Adjustments



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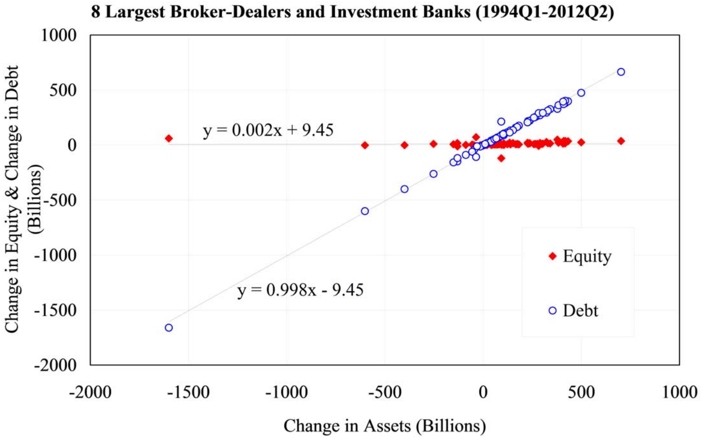
Broker-Dealers and Banks



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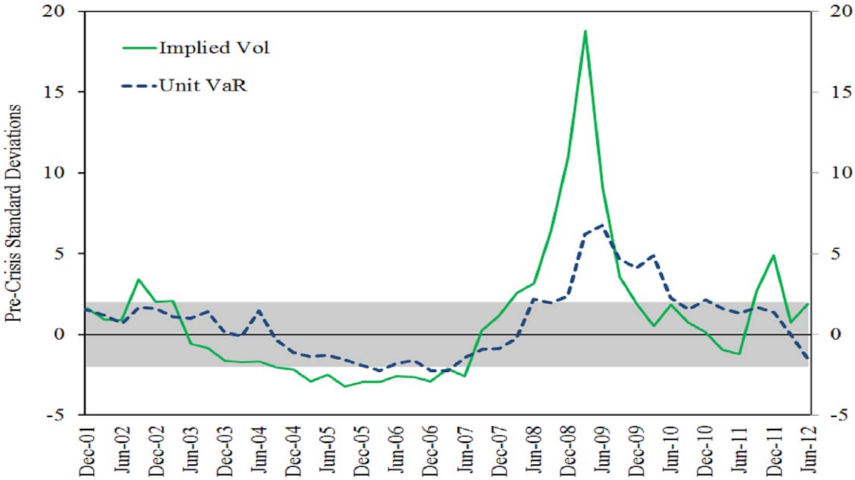
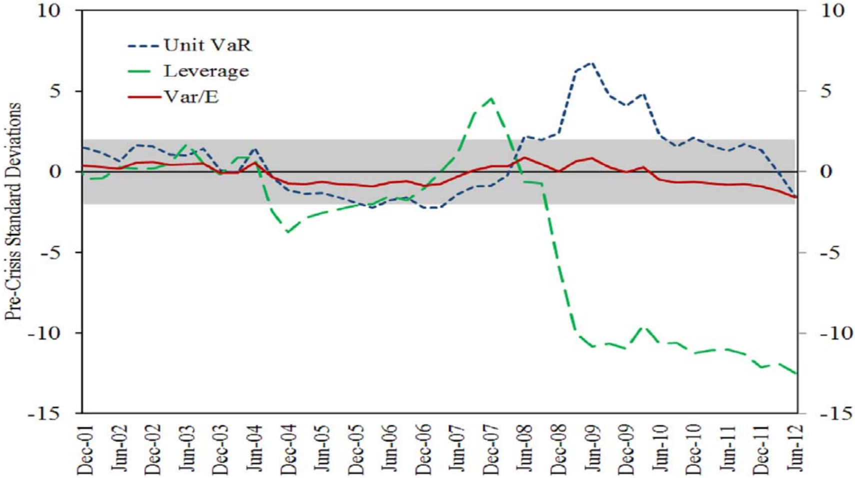
Broker-Dealer *VaR*



Broker-Dealer *VaR*



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Outline

Simulation Parameters

Parameter	Value
$\bar{a}$	0.0651
σ_a	0.388
ρ	0.06
$\rho_h - \sigma_\xi^2/2$	0.05
ϕ_0	0.1
ϕ_1	20
λ_k	0.03
$\rho_{\xi,a}$	0
σ_ξ	0.0388
α	2.5

- Ref.: Brunnermeier and Sannikov (2012)
- Monthly simulation frequency
- 10000 paths; 70 years Back

Equilibrium

Lemma 1

$$\mu_{Rk,t} = \mathcal{K}_0(\omega_t,\theta_t) + \mathcal{K}_a(\omega_t,\theta_t)\sigma_{ka,t} + \sigma_\xi\sqrt{1-\rho_{\xi,a}^2}\sigma_{k\xi,t}$$

$$\mu_{Rb,t} = \mathcal{B}_0(\omega_t,\theta_t) + \mathcal{B}_a(\omega_t,\theta_t)\sigma_{ka,t} + \mathcal{B}_\xi(\omega_t,\theta_t)\sigma_{k\xi,t}$$

$$\mu_{\omega t} = \mathcal{O}_0(\omega_t,\theta_t) + \mathcal{O}_a(\omega_t,\theta_t)\sigma_{ka,t} + \mathcal{O}_\xi(\omega_t,\theta_t)\sigma_{k\xi,t}$$

$$\mu_{\theta t} = \mathcal{S}_0(\omega_t,\theta_t) + \mathcal{S}_a(\omega_t,\theta_t)\sigma_{ka,t} - \mathcal{O}_\xi(\omega_t,\theta_t)\sigma_{k\xi,t}$$

$$r_{ft} = \mathcal{R}_0(\omega_t,\theta_t) + \mathcal{R}_a(\omega_t,\theta_t)\sigma_{ka,t}$$

$$\sigma_{ba,t} = \frac{2\theta_t\omega_t p_{kt} + \beta(1-\omega_t)}{\beta\omega_t(\theta_t-1)}\sigma_a - \frac{2\theta_t\omega_t p_{kt} + \beta(1-\theta_t\omega_t)}{\beta\omega_t(\theta_t-1)}\sigma_{ka,t}$$

$$\sigma_{b\xi,t} = -\frac{2\theta_t\omega_t p_{kt} + \beta(1-\theta_t\omega_t)}{\beta\omega_t(\theta_t-1)}\sigma_{k\xi,t}$$

$$\sigma_{\theta a,t} = -\frac{2\theta_t\omega_t p_{kt} + \beta(1-\omega_t)}{\beta\omega_t}(\sigma_{ka,t}-\sigma_a)$$

$$\sigma_{\theta\xi,t} = -\frac{2\theta_t\omega_t p_{kt} + \beta(1-\omega_t)}{\beta\omega_t}\sigma_{k\xi,t}$$

$$\sigma_{k\xi,t} = -\sqrt{\frac{\theta_t^{-2}}{\alpha^2} - \sigma_{ka,t}^2}$$

$$\sigma_{ka,t} = \frac{\theta_t^{-2}}{\alpha^2} + \sigma_a^2\left(1 + \frac{1-\omega_t}{\omega_t(2\theta_t\omega_t p_{kt} + \beta(1-\omega_t))}\right).$$

Risk-free Rate

- Household Euler equation

$$r_{ft} = \left(\rho_h - \frac{\sigma_\xi^2}{2}\right) + \frac{1}{dt}\mathbb{E}\left[\frac{dc_t}{c_t}\right] - \frac{1}{dt}\mathbb{E}\left[\frac{\langle dc_t\rangle^2}{c_t^2} + \frac{\langle dc_t, d\xi_t\rangle^2}{c_t}\right]$$

- Goods market clearing implies

$$\begin{aligned} dc_t &= d\left(K_t A_t - i_t k_t A_t\right) \\ &= A_t dK_t + \left(K_t - i_t k_t\right) dA_t - A_t k_t di_t - A_t i_t dk_t - k_t \langle di_t, dA_t\rangle \end{aligned}$$

Stress Tests

Inherent limitations to VaR include [. . .] VaR does not estimate potential losses over longer time horizons where moves may be extreme.

Stress Tests

- Could consider a forward-looking capital constraint

$$\theta_t^{-1} \geq \vartheta \sqrt{\mathbb{E}_t \left[\int_t^T \left(\sigma_{ka,s}^2 + \sigma_{k\xi,s}^2 \right) ds \right]}.$$

- Looks like a robust-control constraint
- Rewrite intermediary optimization as

$$V_t(\vartheta) = \max_{\{i,\beta,k,\alpha_s\}} \mathbb{E}_t \left[\int_t^{\tau_D} e^{-\rho(s-t)} w_t(i,\beta,k)\, ds \right]$$

s.t.

$$\frac{\theta_s^{-1}}{\alpha_s} \geq \sqrt{\sigma_{ka,s}^2 + \sigma_{k\xi,s}^2}$$

$$\theta_t^{-1} = \vartheta \sqrt{\mathbb{E}_t \left[\int_t^T \frac{\theta_s^{-2}}{\alpha_s^2} ds \right]}.$$

- "Choose optimal capital plan"

Outline

Intermediaries

Banks
Create new capital; financed by debt issuance to the households

Funds
Hold existing capital; financed by profit sharing contracts with households

- Two types of intermediaries: non-bank ("fund") and bank
- Unit mass of specialists manage funds; unit mass of bankers manage banks
- Future work: interactions between different intermediary types

Fund Sector

- Modeled as in He and Krishnamurthy (2012, 2013)
- Fund is formed each period t as a random match between a specialist and a household
- Specialist contributes all of his wealth w_{ft} to the fund
- Household contributes up to mw_{ft} to the fund
- m: tightness of the specialists' capital constraint
- Specialists control the allocation of fund capital to holding capital projects and risk-free debt

Notice:

- No new capital project creation
- No risky debt

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Specialists' Optimization I

Specialists' maximize expected consumption

$$\max_{\{c_{ft},\theta_{ft}\}} \mathbb{E}\left[\int_0^{+\infty} e^{-\rho t} \log c_{ft} dt\right],$$

subject to the dynamic budget constraint

$$\frac{dw_{ft}}{w_{ft}} = \theta_{ft}\left(dR_{kt} - r_{ft} dt\right) + r_{ft} dt - \frac{c_{ft}}{w_{ft}} dt$$

Specialists' Optimization II

Lemma 2

The specialists consume a constant fraction of their wealth

$$c_{ft} = \rho w_{ft},$$

and allocate the fund's capital as a mean-variance investor

$$\theta_{ft} = \frac{\mu_{Rk,t} - r_{ft}}{\sigma^2_{ka,t} + \sigma^2_{k\xi,t}}.$$

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Banking Sector

- Banks create new capital

$$dk_t = (\Phi(i_t) - \lambda_k)\, k_t dt$$

- Investment carries quadratic adjustment costs (Brunnermeier and Sannikov (2012))

$$\Phi(i_t) = \phi_0 \left(\sqrt{1 + \phi_1 i_t} - 1 \right)$$

- Banks finance investment projects through inside equity and outside risky debt giving the budget constraint

$$p_{kt} A_t k_t = p_{bt} A_t b_t + w_t$$

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Bankers' Optimization I

The representative banker solves

$$\max_{\theta_t, i_t, c_{bt}} \mathbb{E}\left[\int_0^{+\infty} e^{-\rho t} \log c_{bt} dt\right]$$

subject to the dynamic budget constraint

$$\begin{aligned}\frac{dw_t}{w_t} &= \theta_t \left(dR_{kt} - r_{ft} dt + \left(\Phi\left(i_t\right) - \frac{i_t}{p_{kt}}\right) dt\right) \\ &\quad - \left(\theta - 1\right)\left(dR_{bt} - r_{ft} dt\right) + r_{ft} dt - \frac{c_{bt}}{w_t} dt,\end{aligned}$$

and the risk-based capital constraint

$$\theta_t \leq \frac{1}{\alpha\sqrt{\sigma_{ka,t}^2 + \sigma_{k\xi,t}^2}}.$$

Bankers' Optimization II

Lemma 3

The representative banker optimally consumes at rate

$$c_{bt} = \rho w_t$$

and invests in new projects at rate

$$i_t = \frac{1}{\phi_1}\left(\frac{\phi_0^2\phi_1^2}{4}p_{kt}^2 - 1\right).$$

While the capital constraint in not binding, the banking system leverage is

$$\theta_t = \frac{\sigma_{ba,t}^2 - \sigma_{ka,t}\sigma_{ba,t} + \sigma_{b\xi,t}^2 - \sigma_{k\xi,t}\sigma_{b\xi,t} - \left(\mu_{Rb,t} - r_{ft}\right)}{\left(\left(\sigma_{ba,t} - \sigma_{ka,t}\right)^2 + \left(\sigma_{b\xi,t} - \sigma_{k\xi,t}\right)^2\right)} + \frac{\left(\mu_{Rk,t} + \Phi\left(i_t\right) - \frac{i_t}{p_{kt}} - r_{ft}\right)}{\left(\left(\sigma_{ba,t} - \sigma_{ka,t}\right)^2 + \left(\sigma_{b\xi,t} - \sigma_{k\xi,t}\right)^2\right)}.$$

Households

- Household preferences are:

$$\mathbb{E}\left[\int_0^{+\infty} e^{-(\xi_t+\rho_h t)} \log c_t \, dt\right]$$

- Liquidity preference shocks (as in Allen and Gale (1994) and Diamond and Dybvig (1983)) are $\exp(-\xi_t)$

$$d\xi_t = \sigma_\xi dZ_{\xi t}$$

- Households allocate wealth between risky bank debt and contributions to funds

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Households' Optimization I

The representative household solves

$$\max_{\substack{\pi_{kt},\pi_{bt}\\ c_t}} \mathbb{E}\left[\int_0^{+\infty} e^{-\xi_t-\rho_h t}\log c_t dt\right],$$

subject to the dynamic budget constraint

$$\frac{dw_{ht}}{w_{ht}} = \pi_{kt}\theta_{ft}\left(dR_{kt} - r_{ft}dt\right) + \pi_{bt}\left(dR_{bt} - r_{ft}dt\right) + r_{ft}dt - \frac{c_t}{w_{ht}}dt,$$

the skin-in-the-game constraint

$$\pi_{kt}w_{ht} \leq mw_{ft},$$

and no shorting constraints

$$\pi_{kt} \geq 0$$
$$b_{ht} \geq 0.$$

Households' Optimization II

Lemma 4

The households' optimal consumption choice satisfies

$$c_t = \left(\rho_h - \frac{\sigma_\xi^2}{2} \right) w_{ht}.$$

While the households are unconstrained in their wealth allocation, the households' optimal portfolio choice is given by

$$\begin{bmatrix} \pi_{kt} \\ \pi_{bt} \end{bmatrix} = \left(\begin{bmatrix} \theta_{ft}\sigma_{ka,t} & \theta_{ft}\sigma_{k\xi,t} \\ \sigma_{ba,t} & \sigma_{b\xi,t} \end{bmatrix} \begin{bmatrix} \theta_{ft}\sigma_{ka,t} & \sigma_{ba,t} \\ \theta_{ft}\sigma_{k\xi,t} & \sigma_{b\xi,t} \end{bmatrix} \right)^{-1} \begin{bmatrix} \theta_{ft}\left(\mu_{Rk,t} - r_{ft}\right) \\ \mu_{Rb,t} - r_{ft} \end{bmatrix}$$
$$- \begin{bmatrix} \theta_{ft}\sigma_{ka,t} & \sigma_{ba,t} \\ \theta_{ft}\sigma_{k\xi,t} & \sigma_{b\xi,t} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \sigma_\xi \end{bmatrix}.$$

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Equilibrium

An equilibrium in the economy is a set of price processes $\{p_{kt}, p_{bt}, r_{ft}\}_{t\geq 0}$, a set of household decisions $\{\pi_{kt}, b_{ht}, c_t\}_{t\geq 0}$, a set of specialist decisions $\{k_{ft}, c_{ft}\}_{t\geq 0}$, and a set of intermediary decisions $\{k_t, i_t, b_t, c_{bt}\}_{t\geq 0}$ such that the following apply:

1. Taking the price processes, the specialist decisions and the intermediary decisions as given, the household's choices solve the household's optimization problem, subject to the household budget constraint, the no shorting constraints and the skin-in-the-game constraint for the funds.
2. Taking the price processes, the specialist decisions and the household decisions as given, the intermediary's choices solve the intermediary's optimization problem, subject to the intermediary budget constraint, and the regulatory constraint.
3. Taking the price processes, the household decisions and the intermediary decisions as given, the specialist's choices solve the specialist's optimization problem, subject to the specialist budget constraint.
4. The capital market clears at all dates
$$k_t + k_{ft} = K_t.$$
5. The risky bond market clears
$$b_t = b_{ht}.$$
6. The risk-free debt market clears
$$w_t + w_{ft} + w_{ht} = p_{kt} A_t K_t.$$
7. The goods market clears
$$c_t + c_{bt} + c_{ft} + A_t k_t i_t = K_t A_t.$$

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